Managing combined power and heat portfolios in sequential spot power markets under uncertainty

HEMF Working Paper No. 03/2020

by

Andreas Dietrich,

*Christian Furtwängler**

and

Christoph Weber

December 2020

Managing combined power and heat portfolios in sequential spot power markets under uncertainty by Andreas Dietrich, Christian Furtwängler* and Christoph Weber

Abstract

The integrated provision of energy among various energy sectors plays an important role in the process of decarbonisation of large energy systems. An important pillar is thereby the decarbonisation of the heat sector, where nowadays still a large percentage of heat supply originates from high-emission fossil fuels like coal or oil.

In Central Europe, combined heat and power (CHP) plant applications, e.g. in local district heating networks, represent established methods to provide both electricity and heat at the same time, lowering overall fuel demands and lowering concomitant emissions. Heat pumps, converting electricity into heat, are also increasingly adopted by commercial (and household) customers. However, the optimal marketing and production scheduling of the heat and power-providing portfolios under price uncertainty is a challenging and often complex task. The importance of proper uncertainty handling is underscored even more if the optimal dispatch of flexible technologies like storages needs to be considered.

In this paper, we propose an enhanced multi-stage stochastic programming model for coordinated bidding in two sequential markets, namely the one-hour and the fifteen-minute electricity products in the German (day-ahead) spot market.

Our study develops and applies a stochastic mixed-integer linear programming model for a virtual power plant, acting as a price taker in the mentioned electricity markets. The model determines the optimal bidding strategies for a heterogeneous portfolio of small gas-fired motor-CHP units, heat pumps, electric storage heaters and battery storage systems. Thereby, we introduce a novel approach to construct piece-wise linear bidding curves for these markets, choosing their supporting points based on the simulated price paths. For the evaluation of the benefits of decision-making by help of stochastic modelling and optimization with different scenario numbers, we develop a new concept, the Benefit of Stochastic Optimization (BSO) and reflect and contrast our results with the computational burden of stochastic simulation, using the example of a real-world portfolio.

We find that stochastic optimisation is a valuable optimisation method that may inform and improve individual marketer's decision-making processes. However, the observable additional benefits, i.e. compared to deterministic point forecasts, are limited in the investigated cases, while computational expensiveness increases significantly when adding further scenarios.

Keywords : stochastic optimization, combined heat and power, virtual power plant, value of stochastic simulation

JEL-Classification : C32, C61, L94

Christian Furtwängler
(CORRESPONDING AUTHOR)
House of Energy Markets and Finance
University of Duisburg-Essen, Germany
Berliner Platz 6-8, 45127 Essen
+49-(0)201 / 183-6458
Christian.Furtwaengler@uni-due.de
www.hemf.net

Andreas Dietrich
House of Energy Markets and Finance
University of Duisburg-Essen, Germany
Andreas.Dietrich@uni-due.de

Christoph Weber
House of Energy Markets and Finance
University of Duisburg-Essen, Germany
Christoph.Weber@uni-due.de

The authors are solely responsible for the contents which do not necessarily represent the opinion of the House of Energy Markets and Finance.

Content

1 Introduction

The ongoing transformation towards decarbonisation requires the addition of flexible generation and consumption technologies in the different sectors of the energy system. At the same time portfolio owners often struggle to stay profitable in liberalised electricity markets, such as the continental European electricity spot markets – given competitive pressure and prevailing marginal asset pricing (Sen, et al., 2006).

On an everyday basis, this is even more evident for cogeneration units that provide both electricity and heat at the same time. The need for immediate satisfaction of heat demands often induces inflexibilities in electricity generation, resulting in must-run conditions even at unfavourable electricity price levels (Furtwängler & Weber, 2019). Therefore, electricity and heat storage technologies are useful means to decouple heat supply and demand, optimizing contributions generated from heat providing technologies. Additionally, heat pumps, that convert electricity to produce heat, add the possibility to provide heat cheaply at low electricity prices. However, unit commitment and dispatch of such flexible portfolios are not trivial and require sophisticated optimization methods (Havel & Simovic, 2013).

Most notably, optimal trading for flexible power generation and consumption portfolios needs to account for electricity price uncertainty. However, many small portfolio owners like municipalities, who predominantly operate district heating grids and other heat systems, e.g. in swimming baths, often lack resources for elaborate market analysis. For such players, elaborate, but easily replicable methods are needed to capture price uncertainty and enable decision-making in the uncertain market environment. A framework that seems very suitable in this context is stochastic optimization, as it enables decision-making in two- or multi-stage decision problems. As an additional pitfall, however, computation times of resulting stochastic models need to be limited in accordance to real-world market requirements (Dietrich, et al., 2018).

Therefore, this paper aims to answer the following research questions: ***1)*** How can the optimization of small-scale portfolios including power and heat resources be represented by a multi-stage optimization program describing the sequence of short-term Central European electricity markets? ***2)*** How big are the benefits offered by a stochastic representation of price uncertainties if in parallel rigid restrictions such as local heat demands are imposed? ***3)*** What are adequate approaches to obtain coherent and realistic assessments of the benefits of stochastic optimization in such a setting? Furthermore, we are interested in the impact of further installed flexibility on the additional value provided by stochastic optimization.

The paper is structured as follows: After laying out relevant literature on stochastic optimization in auction-based electricity markets with cogeneration in Section 2, Section 3 focuses on the methodology by deriving the stochastic decision structure in the markets under investigation and discussing the methodology of price scenario generation. Furthermore, a novel bidding curve generation method, the developed portfolio optimization model and the enhanced concepts for evaluation of the benefit of stochastic optimization are discussed. Section 4 then describes the characteristics of the real-world portfolio used for evaluation of the developed approach. Section 5 summarizes and discusses the obtained results, giving insight in the additional value of stochastic optimization compared to different benchmarks, such as optimization with perfect forecasts with and without a rolling horizon and deterministic optimization. Finally, Section 6 concludes by answering to the formulated research questions.

2 Literature Review

Deregulated electricity markets and the ongoing shift to electricity production from intermittent wind and solar power production in many countries have spurred the interest in strategies and methods to handle uncertainties within short-term electricity market bidding decisions among power producers and consumers. Accordingly, a broad variety of literature has emerged in this field. A general review on mathematical programming models that capture the optimal bidding problem in day-ahead auction markets is given in Frances & Kwon (2012), regarding stochastic programming models, an overview can be found in Fleten & Kristoffersen (2007) as well as in Boomsma, et al. (2014). Against the background of our paper´s specific portfolio, we identified the following strand of literature that focusses on integrated approaches for optimal CHP plant and heat storage operation planning and bidding in continental European electricity markets:

De Ridder & Claessens (2014) propose a trading strategy for industrial CHP installations selling their power in the Belgian day-ahead and continuous intraday market. Their findings indicate that if bids are made on multiple markets, profits can be increased significantly and, compared to a deterministic approach, a trading strategy based on stochastic dynamic programming may lead to additional gains between 6.5 and 19%. The authors also demonstrate how assumptions about internal power consumption and CHP maintenance cost influence the trading strategy and its monetary outcome. As a shortcoming, correlation between prices on day-ahead and continuous intraday markets is not incorporated here which may overrate additional benefits from multiple market bidding strategies.

Schulz & Werners (2015) develop a model to cope with investment decisions considering heat storage investments for CHP district heating systems operated in the German day-ahead spot market environment. In this paper, focus lies on the determination of the optimal storage capacity that is calculated in the first stage whereas optimal unit commitment is determined in the second stage. Furthermore, different decision makers' risk preferences are considered whereas the bidding process is not modelled and market prices are considered to be given.

Zapata Riveros, et al. (2015) present a two-stage stochastic programming approach to evaluate three different bidding strategies in the Belgian day-ahead spot market for a virtual power plant including renewables and a CHP unit. As a particular feature, the imbalance market is taken into account and a rescheduling of the CHP unit for balancing forecast errors of renewable energy sources is possible within the second optimization stage. For this, piecewise linear shortfall cost functions are used to penalize imbalance volumes. Results from a case study show that the flexibility of a thermal storage can be used to accommodate forecast errors from solar and wind energy effectively and thus, portfolio imbalance costs can be reduced.

In another paper, Han, et al. (2017) develop a multi-stage stochastic optimization model to derive trading strategies for a pool of variable renewables, dispatchable generators, flexible loads and storage devices in sequential day-ahead, intraday and balancing markets. Similar to Zapata Riveros, et al. (2015), their findings indicate that stochastic optimization and coordinated bidding of different units leads to higher flexibility provision, increased expected revenues and lower imbalances. Additionally, effects of risk control are examined here and it is shown that a lower risk exposure leads to decreased trading revenues.

Kumbartzky, et al. (2017) investigate a three-stage stochastic programming problem for short-term production planning of a CHP plant with heat storage, providing district heating. Comparable to our work, the technical conditions of a real-world application are considered within a case study. The suggested methodology illustrates that sequential trading in different short-term electricity markets, namely the German minute reserve and day-ahead spot market, is beneficial for the power plant operator. Regarding CHP plant operation planning and bidding in electricity markets, this paper also gives a comprehensive literature review including deterministic as well as stochastic modelling approaches.

In a recent work, Ackermann, et al. (2019) introduce a two-stage stochastic method to derive stepwise bidding curves for offering strategies in the German day-ahead spot market. The model is applied to a district heating portfolio consisting of multiple CHP units, heat storage, gas boilers and an electric boiler. Their findings, also based on a real-world case study, cannot prove a clear advantage against a "flat-bidding" strategy that uses a deterministic modelling technique. Here, monetary benefits of stochastic optimization turn out small and may even become negative.

A modelling approach rather similar to the multi-stage stochastic mixed-integer linear programming model presented in our paper can be found in Böhringer, et al. (2019). The authors determine optimal trading strategies for a flexible industrial consumer from a demand-side management perspective. For the German secondary control reserve and day-ahead market, a multi-market bidding problem is formulated and applied to a factory´s real load profile. Also, risk preferences are integrated into the optimization problem and it is shown that a risk-averse trading strategy leads to more robust results.

With the exception of Zapata Riveros, et al. (2015) who use quarter hourly time intervals, above mentioned studies are based on an hourly modelling discretisation because trading decisions for the markets under study do not require a shorter time interval. However, some major methodological differences occur regarding price modelling techniques, the number of scenarios used for the stochastic optimization and the bidding curve construction. Furthermore, the number of stochastic variables is diverse because, besides imperfect information about market prices, some bidding problem formulations involve additional sources of uncertainty like electrical or thermal loads or fluctuating renewable production. On the application side, it becomes apparent that the periods under study mainly focus on daily or weekly time horizons. It is also noticeable that most of the reviewed literature focusses on the question of financial improvements from coordinated bidding in a sequential multi market environment under uncertainty, compared to non-coordinated bidding in single markets. However, in many cases the merit of stochastic programming models by itself, compared to deterministic approaches, also known as the value of stochastic solution (VSS), is not quantified[1]. Possible economic advantages that are commonly associated with stochastic optimization methods applied within the energy sector therefore remain unclear. Table 1 summarizes these key characteristics in detail for the papers discussed before.

[1] Note that the concept of VSS has to be rethought if it is to be applied in the context of a backtesting based on historical data as intended here, cf. at the end of this section and section 3.5.

Table 1: Key modelling characteristics of the literature under review

Author(s)/ year	Price modelling technique	Considered markets	Number of price scenarios	Bidding method (curve form)	Period under study	Considered uncertainties	VSS quantified
De Ridder & Claessens (2014)	not specified, based on PDF	DA and ID	not specified	offer stack	one day per month for one year	market prices	yes
Zapata Riveros, et al. (2015)	Non-parametric probabilistic approach	DA and IM	10 for DA and for IM, in first stage, then 50 for IM	not specified, seems to be single-bid	three seasonal weeks	market prices, RE production	no
Schulz & Werners (2015)	not specified, apparently historical price paths	DA	8 for DA	not specified, seems to be single-bid	one week for unit commitment	market prices, power and heat demand	no
Kumbartzky, et al. (2017)	SARIMA	DA and RM	5 for DA, 10 for RM	not specified, seems to be single-bid	five days	market prices	no
Han, et al. (2017)	not specified	DA and ID	6 for DA, 6 for ID	scenario bidding constraints	one day	market prices, RE production	no
Ackermann, et al. (2019)	Autoregressive Exogeneous (ARX) models	DA	100 for DA	piecewise linear curve vs. single-bid	four seasonal month	market prices, heat demand	yes
Böhringer et al. (2019).	Monte Carlo simulation	DA and RM	not specified	scenario bidding constraints	one week	market prices	no

Abbreviations: DA: day-ahead market, ID: intraday market, IM: imbalance market, RM: reserve market, PDF: probability density function, RE: renewable energies, VSS: value of stochastic solution

Our approach presented below goes beyond the previously discussed optimal bidding models for short-term trading of cogeneration units and virtual power plants in at least four aspects that, to the best of our knowledge, have not been considered in literature so far.

First, we present a methodology for coordinated bidding into a sequence of hourly and quarter hourly day-ahead spot markets. In Germany and other continental European countries, trading in quarter-hourly market segments has become more attractive for electricity companies with flexible production and/or consumption portfolios due to higher price variability. Therefore it is of special interest to analyse potential benefits as well as possible drawbacks of stochastic optimization in this market environment.

Second, for both markets, we also account for the specific bidding rules notably in terms of piecewise linear bidding curves. The construction of bidding curves is a key question within the optimal bidding problem and, until now, piecewise linear bidding curves have been modelled as a simplified representation only. Consequently, optimal bid solutions are not guaranteed[2]. Therefore, our work introduces a novel approach to construct piecewise linear bidding curves by the definition of fixed supporting points for each hourly and quarter-hourly bidding curve.

Third, the portfolio in our real-world case study exhibits a broad spectrum of flexibility options. Small CHP-units, heat-pumps, thermal storages, night storage heating and battery storage systems are believed to become important flexibility providers in the German electricity system. However, it is unclear if stochastic optimization models that account for technological diversity as well as for large numbers of units and individual heating demands are suitable for practical application. Within this context, this paper also addresses the problem of computation time.

Finally, we believe that the use of type days/weeks/months may lead to imprecise findings regarding financial advantages of stochastic optimization techniques. For this, we extend the period under study to one full year and we also quantify the value of the stochastic modelling approach compared to a deterministic one explicitly. We therefore develop and apply a new concept, called the Benefit of Stochastic Optimization with a Rolling Horizon (BSO_n^{RH}, n denoting the number of scenarios), which is suited to determine the benefits of stochastic modelling in a backtesting framework. This concept is explained in detail in section 3.5.

3 Methodology

In this section, we describe the methodological approaches applied within our case study. Subsection 3.1 gives an overview of the considered electricity spot market environment, relevant for the marketing decisions of the stochastic program decision structure, presented in subsection 3.2. Subsection 3.3 explains the stochastic electricity spot price modelling and the methodology utilized to generate the price scenarios that are applied in the stochastic portfolio optimization program, subsequently described in subsection 3.4. In subsection 3.5, the backtesting and its associated evaluation concepts for the benefits of stochastic representations of optimization problems under uncertainty used in Section 5 of this paper are introduced. Additionally, Appendix 1 gives insight into the model equations, including the mathematical transformation of the most relevant technical characteristics of the portfolio. An overall description of the real-world portfolio under study and its specific unit features is presented separately, see Section 4.

[2] Bidding curve constraints as modelled in Han et al. (2017), Ackermann et al. (2019) and Böhringer et al. (2019) are not based on supporting points. Here, if the market price realisation falls between the price scenarios, optimal production or consumption schedules are not defined.

3.1 Relevant marketing decisions with a sequence of day-ahead spot market auctions

In our study, we assume the portfolio owner to bid power supply and/or demand into two subsequent electricity spot markets. Such a sequence has notably been established in Germany, namely the EPEXSPOT day-ahead (DA) auction[3], where electricity products with highest time-resolution are hourly products, and the EPEXSPOT intraday opening (IDO) auction, which represents the first opportunity to trade quarter-hourly products. Recently, quarter hourly products and corresponding auctions have also been introduced in other continental European countries such as France, Belgium, the Netherlands and Austria (EPEXSPOT, 2020). To reduce the complexity of the resulting bidding problem, we neglect all up-front long-term delivery contracts as well as block products, day base and day peak products[4].

For both, DA and IDO auctions, market participants have to submit their power bids in advance by means of a supply curve, consisting of a price/quantity combination for each product. Notably, bidders need to define a finite number (up to 256) of electricity spot price levels as supporting points for their price/quantity curves. These price levels, serving as the supporting points of the bidding curve, cannot be altered for different hours of the day. Between these supporting points, bid amounts are calculated by linearization, resulting in a piecewise-linear bidding curve for each considered product. It is required that these curves are monotonic increasing, but not necessarily strictly monotonic increasing, i.e. it is possible to assign the same marketed quantity to various neighbouring price levels (EPEXSPOT, 2019).

For the German market, the sequence of market decisions is as follows: First, the DA auction is held daily on 12 am for the following day. Results of the DA auction are published between 12:55 am and 1:50 pm. Therefore, the quantities allocated in the DA auction are known to market participants before they place their bids for the subsequent IDO auction on 3 pm. The results of this auction are published as soon as possible from 3:15 pm (EPEXSPOT, 2019). After the results of this auction are published, continuous intraday trading of both hourly and quarter-hourly products is possible. However, we assume that small market participants, managing small portfolios, will not be acting in this market due to a lack of dedicated resources. Instead, the resulting quantities of the DA auction and IDO auction will be considered as fixed hourly and quarter-hourly trading positions in unit commitment and dispatch optimization for the following day.

---

[3] The German DA auction also covers the market area of Luxembourg. Until 30th September, 2018, also Austria was part of this common market.

[4] Day Base product is referring to a 24h-block containing all hours of one day, Day Peak product to hours 8-20. On July 1st, 2019, there exist 17 further block products on the EPEXSPOT Day-Ahead market, as well as 6 further block products of the EPEXSPOT Intraday Auction, bundling different hours and quarter-hours of the day (EPEXSPOT, 2019).

3.2 Stochastic Decision Structure

According to this market environment, the sequence of implemented stochastic programs in this paper consists of three subsequent optimizations:

1. Optimization of the day-ahead bids represented by jointly submitted hourly bidding curves (set of hourly bidding curves)
2. Optimization of the bids for the intraday opening auction, represented as set of quarter-hourly bidding curves given the revealed actual DA auction outcome and
3. Optimization of asset operation (unit commitment and dispatch) given both the revealed DA and IDO auction outcomes.

Figure 1 shows an exemplary piece-wise linear bidding curve and how the traded volumes for each individual hour and quarter-hour are determined therefrom based on the realized market prices. The bidder's individual piece-wise-linear bidding function is described by the (optimized) coordinates of supporting points of the bidding curve (denoted here by the bid price $P_{h,ih}^{DA,lb}$ and the bid quantity $p_{h,ih}^{DA,lb}$ at the left boundary of the bidding curve interval ih for an hourly product h) and evaluating the bidding function at the realized spot price provides the marketed amounts.

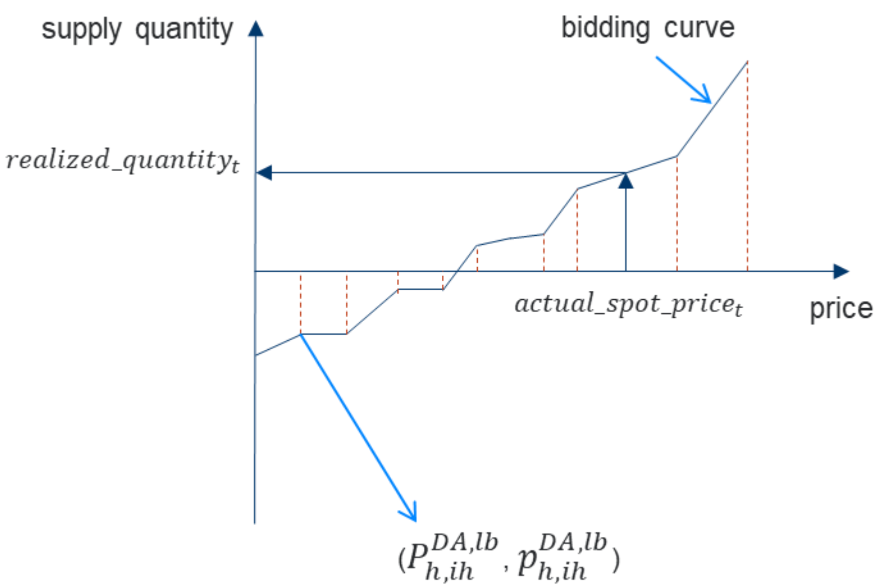

Figure 1: Bidding curve and identification of the allocated quantity with realized spot price.

The flow chart describing the sequence of market decisions and their corresponding optimization runs can be found in Figure 2. The overall marketing problem thus consists of three optimization runs, with new information revealed after the first and second run.

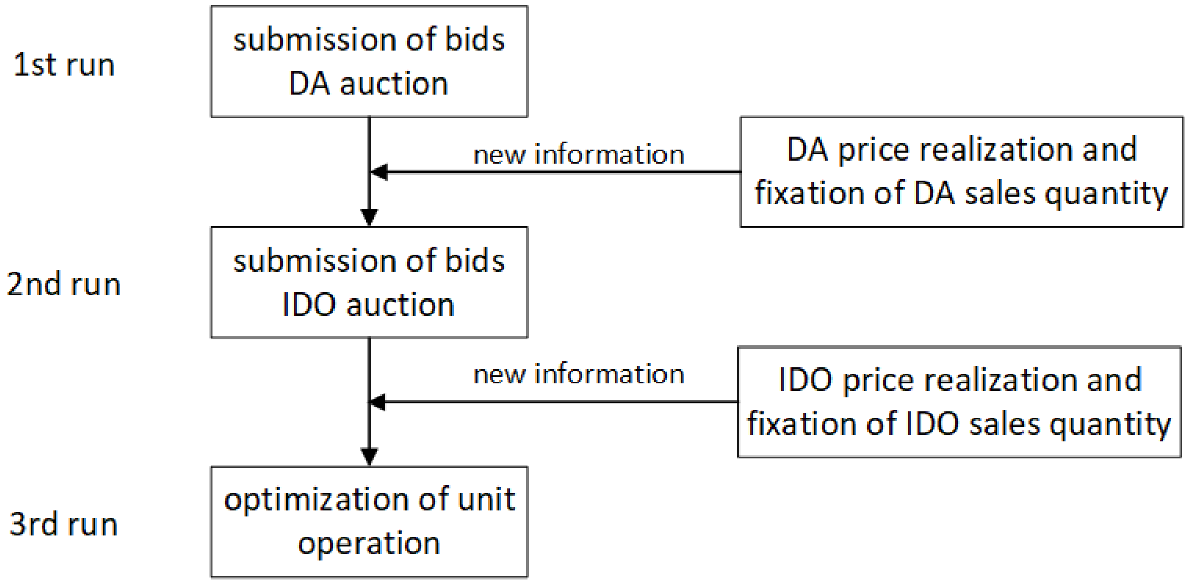

Figure 2: Flow chart of the sequence of the implemented programs.

The stochastic structure for the optimization runs is depicted in Figure 3 for the first and the second optimization run, and in Figure 4 for the third. The optimization also takes the second half of the auction day itself into account, as the asset dispatch after noon of the auction day might still be adjusted by the asset operator. As all price and sales quantity information for these twelve hours is already known, these hours are part of the deterministic root node of the scenario tree. The hours and quarter-hours of day 2 are belonging to the hourly and quarter-hourly products whose bidding curves are being optimized[5]. Here, n market price scenarios are utilized for different possible market outcomes. This way, marketed quantities for a variety of different price developments are identified and inform the decision on the bidding curves to be submitted. The methodology of price scenario generation is discussed in section 3.3 of this paper. Day 3 is also explicitly considered with bidding curves of its own such that the plant states and storage level management at the transition between days 2 and 3 are accurately represented.

[5] Day 2 denotes the day ahead for which the sales quantities are decided upon by the auctions staged on day 1.

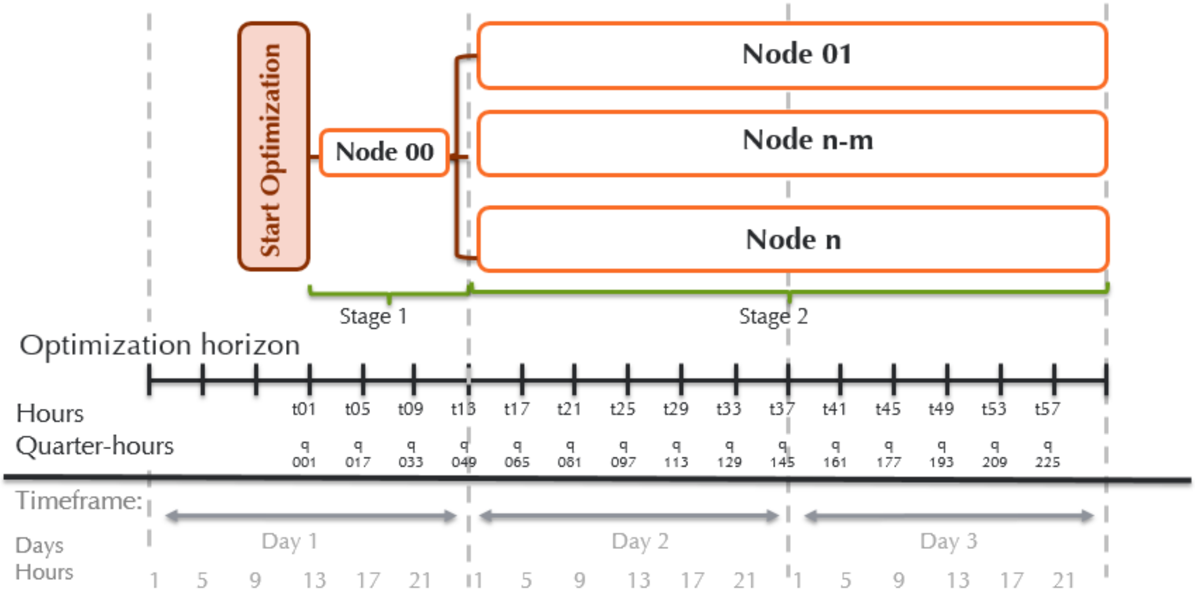

Figure 3: Stochastic tree structure of 1st and 2nd optimization.

After the second optimization, marketed quantities have been fixed for both hourly and quarter-hourly products of day 2. During dispatch optimization, the deterministic root node therefore extends to these hours and quarter-hours, as depicted in Figure 4.

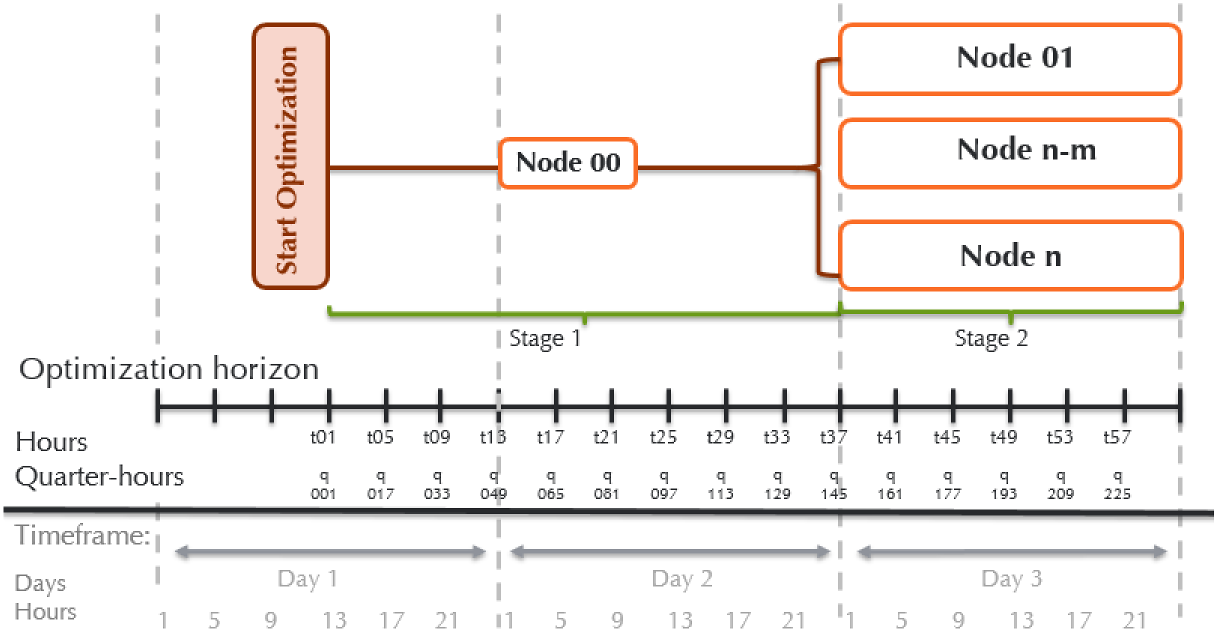

Figure 4: Stochastic tree structure of 3rd optimization.

This three-stage optimization procedure is repeated for each day of the backtesting period. The corresponding rolling planning application including the transition between days is described in more detail in Section 3.5.

3.3 Modelling of market price uncertainty and determination of price scenarios

To obtain a stochastic representation of the electricity price as an uncertain variable from which scenarios may be derived, we apply the approach of Pape, et al. (2017), who investigate the dynamics of hourly electricity prices of the German DA auction as a panel of 24 cross-sectional hours. Their approach may be summarized as follows:

(i) Treat the time series of spot prices as panel of individual hours and transform the original values by an adjusted log-transformation.

(ii) Determine the main deterministic influences and the residuals by means of an OLS (Ordinary Least Squared) regression.

(iii) Map the residuals' empirical distribution to a normal distribution; respectively map the empirical cumulative distribution function of transformed prices to the inverse of the cumulative standard normal distribution.

(iv) Identify common factors of hourly prices by means of a Principal Component Analysis (PCA).

(v) Model lagged effects of price level and price volatility by an ARMA(1,1)-GARCH(1,1) process.

(vi) Use a rolling window technique to estimate prices using the last 173 days as calibration period.

(vii) For each day, use Monte Carlo Simulation to generate 1000 independent price paths.

In this paper, the exact same approach is applied to model the *quarter-hourly* products of the IDO auction instead of the hourly DA auction prices. Subsequently, a standard k-means algorithm[6] is used to reduce the number of scenarios, with k denoting the number of resulting scenarios after reduction. We choose k=1 for a deterministic price forecasting reference[7], and higher k's for stochastic representations to introduce in our stochastic optimization model, described in sections 3.2 and 3.4. The probabilities of the generated scenarios result as the number of scenarios of the original Monte Carlo simulation that are added to the respective clusters, divided by the total number of Monte Carlo simulations.

---

[6] The k-means clustering concept was first developed by MacQueen (1967), the standard algorithm solving the k-means problem was developed in Lloyd (1982). In our analysis, we use the standard k-means function provided by MATLAB version R2019a.

[7] A deterministic price forecasting reference is needed to determine the value added by solving the stochastic representation of the optimization problem in contrast to just modelling uncertainties by means of point forecasts. The underlying concepts, differentiating between optimization problem representations with and without a rolling horizon, as well as the proposed evaluation concepts of the Expected Value of Perfect Information with a Rolling Horizon ($EVPI^{RH}$) and the Benefit of Stochastic Optimization with a Rolling Horizon and n scenarios (BSO_n^{RH}) are explained in detail in Section 3.5.

To capture the effects of unusual price developments that are often eliminated during scenario reduction and to achieve a higher coverage of the underlying price distribution by the reduced scenarios, for the stochastic cases (k>1), we manually add two extreme price scenarios. They contain the historic highest and lowest prices, measured for the respective quarter-hourly product.

Finally, the average of four quarter-hourly products belonging to one hour is calculated and defined as the price scenario forecast for the respective hourly products of the DA auction. This solves arbitrage issues between hourly and quarter-hourly products during the 1st optimization that would arise if both products were modelled separately. More precisely, if the mean of the quarter-hours does not equal the hourly price, an optimization model will be able to generate (possibly infinite) additional gains from buying the product(s) with the lower price and re-selling the other product(s) covering the same timeframe at the higher price[8], unless measures to prevent arbitrage are undertaken in the optimization model. Those, however, may again cause undesired side-effects[9].

The lack of individual modelling of hourly products to prevent arbitrage-related issues leads to a lack of prediction accuracy for the hourly products in the DA auction, as for example information sets differ due to the time lag of three hours between these two auctions on the same day. This effect, however, is very limited if this approach is compared to a separate modelling of Day-Ahead and Intraday opening auction prices. A short analysis of forecast accuracy parameters for both products, if modelled jointly or separately, can be found in Table 2. The model on average slightly underestimates both DA and IDO prices and struggles to capture the magnitude of price deviations of the first (denoted as qh1) and last quarter-hour (denoted as qh4) from the mean price of the corresponding hour. The approach performs better for hourly products than for quarter-hourly products. The overall price accuracy, however, is reasonably good compared to other time-series-based modelling approaches. The most obvious example applying the same method is the above-mentioned original paper by Pape, et al. (2017), where the authors apply various variations of this approach to 2015 price data,

---

[8] For example, if only the sum of both trading positions is restricted by the minimum and maximum production and consumption of the underlying portfolio, the model is unbounded, if prices are not arbitrage-free. If the marketed quantities in both markets are each restricted by the minimum and maximum production and consumption of the underlying portfolio, the problem is not unbounded, but there still is room for arbitrage up to said amounts.

[9] As a very basic example, if we do not allow negative sales (buying) to the individual markets at all, we restrict our possibilities in trading on both markets, as we are not able to react to individual quarter-hourly prices lower than our marginal cost of electricity production. Ideally, the optimization model wants to re-buy said amounts previously sold for the corresponding hourly product (assuming said price for the hourly product exceeded our marginal cost). Therefore, from the point of view of the optimization model, it would often be a better strategy to "wait" for the product with the highest time resolution and not market the hourly product at all – which leads to an unnecessary restriction of the solution space and may exclude the optimal solution (without arbitrage) of the original problem.

obtaining a MAE between 4.06 and 4.25 €/MWh for different specifications of their estimation model for the DA auction prices. While there are no papers covering quarter-hourly forecasts of the same market for the exact same timeframe (01.10.2016-30.09.2017), Kath & Ziel (2018) test a similar standard forecasting method on a training period from 08.10.2015 to 06.10.2016 (364 days) and create forecasts for the period between 07.10.2016 and 31.05.2017, partly using additional information from the Swiss day-ahead auction taking place two hours ahead of the EPEXSPOT DA auction. In this setting, simple OLS optimization including dummy variables for some days of the week and additional fundamental wind and load information leads to MAE values of 4.26 €/MWh, respectively 4.17 €/MWh if information about Swiss auction results is included. However, without the prior elimination of selected dummy variables, this approach yields MAE values of over 7 €/MWh. Kath & Ziel (2018) then enhance their results by making use of a combined lasso and ridge regression, lowering MAE values below 4 €/MWh. Given that the approach by Pape, et al. (2017) uses only prices and weekday dummies as explanatory variables, however, the still considerable performance results suggest that this simpler and less data-reliant approach should be more adequate for the given research question of this paper.

Table 2: Prediction accuracy of chosen price modelling approach.

	Mean Error	**Mean Absolute Error**				
	[€/MWh]	[€/MWh]				
Modelled prices		Ø	qh1	qh2	qh3	qh4
IDO auction	-0.66	5.7	6.18	5.35	5.33	6.13
DA auction (as mean of IDO auction)	-0.69	4.7	-	-	-	-
DA auction (modelled separately)	-0.26	4.6	-	-	-	-

Figure 5 shows the resulting quarter-hourly price scenarios with k=60 for a given two-day period in the dataset without the manually added extreme price scenarios. Figure 6 depicts matching hourly price scenarios. The first 12 hours, respectively 48 quarter-hours thereby represent prices already known from the previous auction rounds as described in the previous subsection. As can be seen, the characteristic “sawteeth” structure of quarter-hourly German IDO auction prices (Pape, et al., 2016) (Pape, et al., 2017) is replicated quite well by the chosen model.

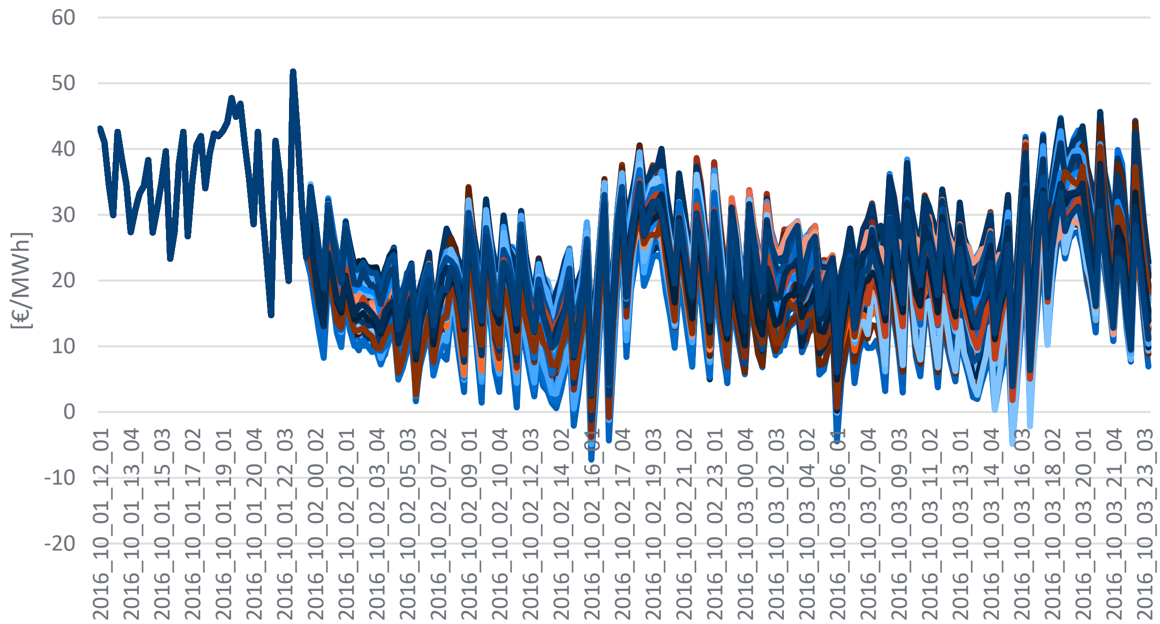

Figure 5: Quarter-hourly price scenarios for example day in autumn 2016 (without extreme price scenarios)

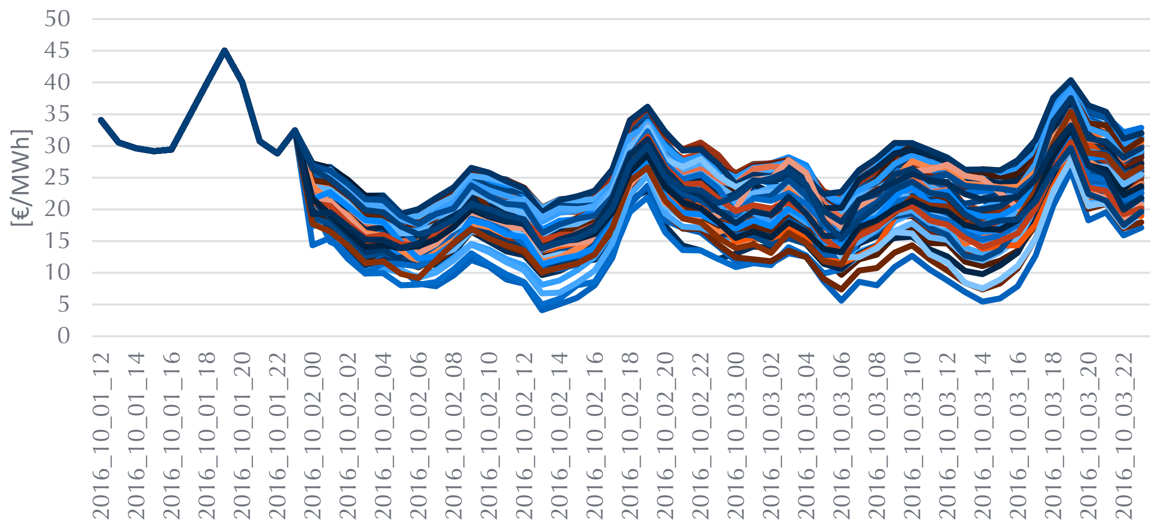

Figure 6: Hourly price scenarios for example day in autumn 2016 (without extreme price scenarios)

3.4 Portfolio and Bidding Model

The price scenario paths are fed into a mixed-integer linear optimization model for each optimization stage (MILP). This model extends earlier work in the dissertation thesis of Kempgens (2018) by the consideration of quarter-hourly products and further technical restrictions, i.e. including the modelling of heat storages and heat pumps. Contrary to Kempgens (2018), the provision of balancing power is not considered in our approach, because the portfolio assets (cf. section 4) do not meet prequalification criteria for participation in the German reserve markets, due to their size.[10]

The optimization model implements a unit commitment model, including the majority of the constraints enumerated by Kempgens (2018). Her model in turn is an extension of the models described by Thorin, et al., (2005), Brand & Weber (2005) and Woll & Weber (2006). The comprehensive model description can be found in the Appendix of this paper.

The objective function of the portfolio optimization problem corresponds to the difference of realized revenues and operational cost. Revenues come from the two mentioned electricity spot markets (day-ahead and intraday opening auction) and, for some CHP units also from additional support mechanisms such as the German Renewable Energy Act ("Erneuerbare-Energien-Gesetz", EEG). Depending on the flexibilities in place, operational cost arise from fuel cost in case of gas-fired CHP systems and gas boilers, cost for alternative district heating supply or from expenses for power-consumption of the electric heating systems and for battery charging. Within this context, it is noteworthy that only the aforementioned spot market prices are included in the cost for electricity consumption or battery charging. Additional cost components arising from grid fees, taxes or other regulated electricity price components are left out of consideration.

The market outcomes of the DA and IDO auctions, i.e. the marketed quantities, are calculated on the basis of the realized spot prices by evaluating the bidding curves obtained in the previous optimization before starting the next optimization run, as described in Section 3.2.

The supporting price points for the hourly and quarter-hourly bidding curves are determined based on a clustering of all simulated electricity prices. These are clustered using a k-means algorithm, with k corresponding to the number of intervals of the resulting bidding curves, both for the quarter-hourly and the hourly prices. The bidding curves' interval boundaries are computed as the mean of the highest price realisation p_n^{max} in price cluster n and the lowest price realisation p_{n+1}^{min} of the neighbouring price cluster $n+1$ with $c_{n+1} > c_n$ for the centroid values c_n and c_{n+1} of the two clusters (cf. Figure 7). This results in $N-1$ boundaries for N bidding curve intervals.

[10] Minimum reserve bid sizes in the years 2016 and 2017 amounted to 1 MW for Frequency Containment Reserve (FCR) (Bundesnetzagentur, 2011), respectively 5 MW for automatic and manual Frequency Restoration Reserve (FRR-a, FRR-m) (Bundesnetzagentur, 2011) (Bundesnetzagentur, 2011).

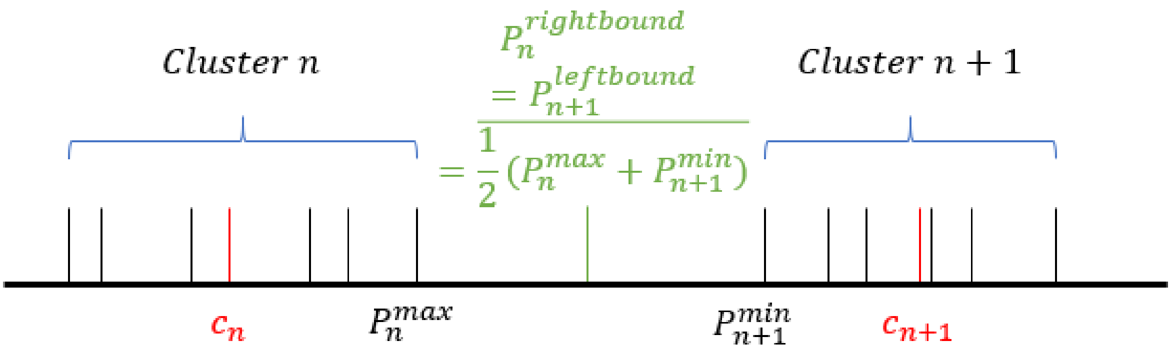

Figure 7: Interval boundary calculation between two price clusters.

The left boundary of the lowest and the right boundary of the highest interval need to be set manually. The chosen parameters are listed at the beginning of Section 5. Finally, each price realisation within the simulated spot market price paths is assigned to the respective resulting price interval it is located in.

3.5 Evaluating the merits of stochastic programs with rolling horizon in a backtesting approach

The backtesting described in this subsection aims to assess the benefits of a stochastic representation of uncertain parameters along with a corresponding stochastic optimization compared to a deterministic uncertainty modelling and optimization approach based on a sufficiently long sequence of observations.

The stochastic program described in the previous sections is therefore applied for a whole year in said backtesting approach, with the year in this case ranging from October 1st 2016 to September 30th 2017. The sequence of optimizations, information updates and parameter fixations are described in Figure 8 below. For each day in the investigated year, three-optimization runs as described in section 3.2 are carried out.

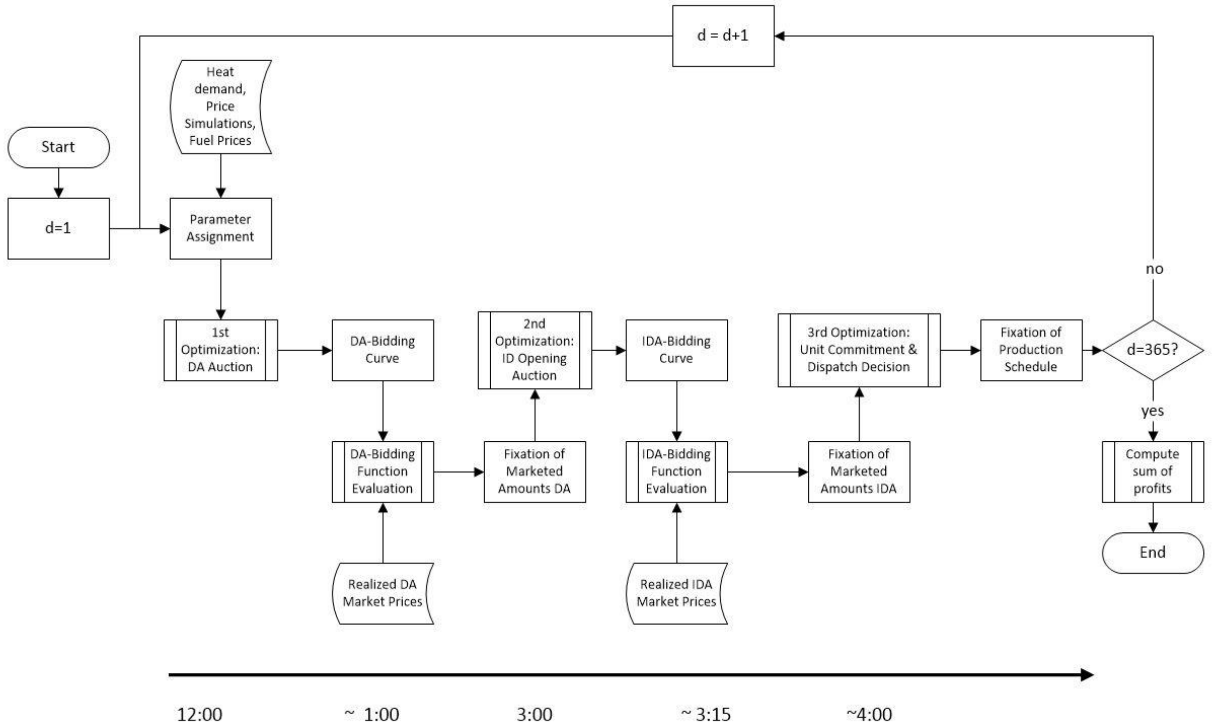

Figure 8: Flow chart of the sequence of stochastic programs during a rolling-horizon backtesting.

After the first and second daily optimization, the resulting bidding curve is evaluated using the actual historical price realisations of the respective time-periods, following the clearing mechanism described in Figure 1 (cf. Section 3.2). The determined allocated quantities after the first and second optimization remain fixed and thus enter all following optimizations of the same day as a parameter.

After the 3rd daily optimization (unit commitment and dispatch, see Subsection 3.2), trading and operation decisions for the first 24 hours/96 quarter-hours including storage levels are saved, with the storage levels after quarter-hour 11:45-12:00 on day 2 serving as starting storage levels for the subsequent optimization period[11]. Also the trading results for the hours 25 to 36 (and the corresponding quarter hours) are saved and are taken as given in the next daily iteration loop, i.e. for $d+1$. From $d=2$ onwards, the sold electricity quantities for the first 12 hours/48 quarter-hours are hence known from the auctions held on the previous day and remain fixed.

[11]The transition between subsequent auction days is realized by saving all marketing and dispatch decisions from noon of day 1 until noon of day 2 (t01-t24, respectively q001-q096) after the third optimization.
The marketed electricity amounts from both the DA and ID Opening Auction from noon of day 2 until midnight of day 2 (t25-t36, respectively q097-q144) remain fixed. The time-horizon subsequently advances by 24 hours such that for example hour 25 becomes hour 1 of the next iteration and the sequence starts again, with former day 2 becoming day 1, day 3 becoming day 2, and so on.

Storage levels at the end of the 60-hour optimization period are assumed to be zero for every optimization period $d \in \{1, \dots, 365\}$[12].

The focus of the evaluation provided in Section 5 is on the additional value generated for the decision maker (in our case a small power and heat portfolio owner) by stochastic representation of the three-step electricity marketing decision problem described in Section 3.2. Therefore, concepts for assessment of decision-taking under uncertainty are needed, considering the described rolling-window approach and both deterministic and stochastic problem representations. We thus propose the following concepts and metrics for our following analysis[13]:

In a world without uncertainty, that means a world with **perfect information**, perfect decisions can be taken. There is no need for representation of uncertainty, and the realised values of otherwise uncertain parameters may enter an optimization problem directly. Thus, there is no need for a non-deterministic representation of this optimization problem, either. However, in the problem investigated in this paper, we need to differentiate between **perfect information with (PI^{RH}) and without a rolling horizon (PI)**. Under perfect information, a rolling horizon approach such as the one described above, will always be outperformed by a full intertemporal optimization, as not all intertemporal dependencies between subsequent optimization days, i.e. stemming from different optimal storage operation decisions are accounted for within each of the single optimization runs with limited planning horizon. Hence the outcomes in a sequence of daily optimizations will be different from the results of an (integral) yearly optimization without consideration of a rolling horizon. This happens although in both cases perfect information about the parameters (prices) is available.

[12] At the beginning of the very first optimization period of the backtesting ($d = 1$), storages are assumed to be charged at 50% of their maximum filling level with all units being switched off (no traded amount in the first 12 hours, respectively 48 quarter-hours). In these 12 hours, heat demand is also assumed to be zero to ensure feasibility.

For the storage level at the end of the 60-hour optimization period, no final value is assigned as such value is difficult to anticipate. Yet the extension of the planning period to 60 hours should suffice to eliminate major end-of-horizon effects for the considered small storages.

[13] The concepts and evaluation metrics introduced here are by our understanding related to, but different from the concepts and metrics established by Birge & Louveaux (2011), namely concepts such as the Expected Value of Perfect Information (EVPI) and the Value of Stochastic Solution (VSS, cf. Literature Review in Section 2), that are often applied within stochastic optimization contexts. We find our own definition for two reasons: Firstly, these concepts were defined and applied for a specific problem type (recourse problems, i.e. problems, where decisions may be adjusted at a later stage) and in a setting without consideration of a rolling-horizon and decision variables with time-interdependencies in between said different time horizons. Secondly, we take a different perspective, as we are interested in the long-term benefits of a short-term decision-making approach by help of stochastic optimization, rather than focusing on an objective function value increase for a one-time decision. Therefore, we define our parameters directly on our backtesting results.

In the following, we analyse the outcome in the rolling-horizon cases by means of an ex-post computation of the resulting "objective function value" of the whole year, after all daily optimizations are completed. Note that this value does not refer to the optimized objective value of a *single yearly optimization* but is the result of a *multitude of daily optimizations* whose results are merged to an annual schedule after the last optimization is completed.

In the result section of this paper, both perfect information outcomes are stated – however, the outcome in the case of $\boldsymbol{PI^{RH}}$ is a more appropriate benchmark when evaluating the quality of daily decision taking under uncertainty (with and without stochastic representation), as the differences in the outcomes between cases with uncertainty and the $\boldsymbol{PI}$ case contain both the effects of parameter realisation uncertainty and myopic decision-making[14].

In our case, the PI^{RH} solution can be computed by solving all three daily optimization runs with real market prices instead of price simulations. In contrast, the PI solution of the yearly decision problem can be identified by solving a single deterministic optimization given real market prices over all time-steps of a year with simultaneous optimization over all operation variables and disregarding the bidding curves.

In the real-world decision-making situation of a small portfolio holder, perfect information does not exist, but **uncertain realizations** of prices or other parameters need to be accounted for in marketing decisions, and thus need to enter the daily optimization model. The straightforward way to identify the (ex-ante) optimal marketing decision is to solve the **expected value problem with rolling horizon** (EV^{RH}). Here the expected value (forecast) of each uncertain parameter is entered into a **deterministic optimization model**. In the case investigated in this paper, this is the expected electricity price path forecast resulting from the price path modelling approach described in Section 3.3[15]. The value added by perfect information (respectively lost by not

[14] One should note that this myopic decision-making is by no means the result of a faulty planning process or market design, but a necessity. Short-term uncertainties in the energy markets, including, but not limited to, short-term changes in electric load and renewable production, or heat demand uncertainties, are creating the need for short-term energy planning and action in the respective markets. Modelling the sequence of all short-term electricity markets that are included in this paper while representing uncertainty for a whole-year in a joint multi-stage optimization problem would result in a problem with more than 1,000 stages and would not be solvable due to the curse of dimensionality and the resulting computational expensiveness. Therefore, a rolling horizon approach such as the approach described here is needed.

[15] To be precise, the price paths entering the optimization code are in this case created by applying the k-means algorithm on the 1000 Monte Carlo simulations of price paths, using k=1. The resulting price path thus represents the path of average price realizations over all simulated price paths, rather than the out-of-sample forecast values of the time-series forecast of each product. However, there is virtually no difference between these two tuples due to the high number of Monte Carlo simulations entering the k-means algorithm.

having perfect information) is in the following defined as expected value of perfect information with a rolling horizon ($EVPI^{RH}$) [16]:

$$EVPI^{RH} = |objective(PI^{RH}) - objective(EV^{RH})| \quad (1)$$

This value can furthermore be used as benchmark for stochastic representations of the optimization problem under uncertainty. If the uncertainty of parameter realizations is modelled by scenarios with an assigned probability, we can solve the corresponding **stochastic optimization** problem.

For the stochastic optimization problem at hand, the modelled uncertain parameters are the prices. The optimized bidding curve in the stochastic case takes additional price realization possibilities (modelled accordingly to the historical price distribution) per time-step into account, leading to a more differentiated, and thus better suited piece-wise linear bidding curve.[17].It is possible to compute the annual outcome of these daily **stochastic optimizations (SO)**. We define $\boldsymbol{SO_n^{RH}}$ as **the stochastic optimization problem with a rolling-horizon and $\boldsymbol{n}$ scenarios.** The so called **benefit of stochastic optimization with a rolling horizon and $\boldsymbol{n}$ scenarios ($\boldsymbol{BSO_n^{RH}}$)** is then obtained by subtracting the annual outcomes of the stochastic problem with n scenarios and the EV^{RH} problem[18]:

$$BSO_n^{RH} = objective(SO_n^{RH}) - objective(EV^{RH}) \quad (2)$$

Note that the benefits of stochastic optimization, as defined here, depend not only on the stochastic optimization (and the corresponding number of scenarios), but also on the quality of the probabilistic price forecasts and the appropriateness of the scenario reduction approach applied to the original Monte Carlo price simulations. A detailed evaluation of the impact of these algorithms is yet beyond the scope of this paper.

4 Characteristics of the portfolio under study

This section describes the key technical and economical characteristics of the portfolio considered within our case study. The portfolio mainly consists of units which supply heat to municipal or residential uses. These encompass motor-based CHP systems and electric heating

[16] The problem investigated in this paper is a maximization problem, therefore $EVPI^{RH}$ is always greater than or equal to zero, even without taking the absolute value of the difference $objective(PI^{RH}) - objective(EV^{RH})$. However, by using the absolute value in this definition, the formula may also be directly applied to the case of a minimization problem.

[17] As a result, the stochastic optimization is expected to yield a different, better outcome than the deterministic case, EV^{RH}.

As noted above (cf. Footnote 15), EV^{RH} is obtained by applying the k-means clustering with k=1 to the Monte Carlo price simulations from our underlying quarter-hourly electricity price distribution. For k>1, stochastic scenario price paths are created that we can enter into the optimization model.

[18] By design, $SO_1^{RH} = EV^{RH}$, so $BSO_1^{RH} = 0$.

systems like heat pumps and electric storage heaters. As an additional component, a battery storage system is considered. Figure 9 provides an overview of the studied case. We base the portfolio on actual units analysed in the field test of the research project "Stadt als Speicher" ("city as a storage")[19]. Therefore, specific technical asset data and heat demands are used. A more detailed description is given in Dietrich, et al. (2018) and Dietrich (2020). The formal implementation of the unit characteristics and constraints is described in the Appendix to this paper.

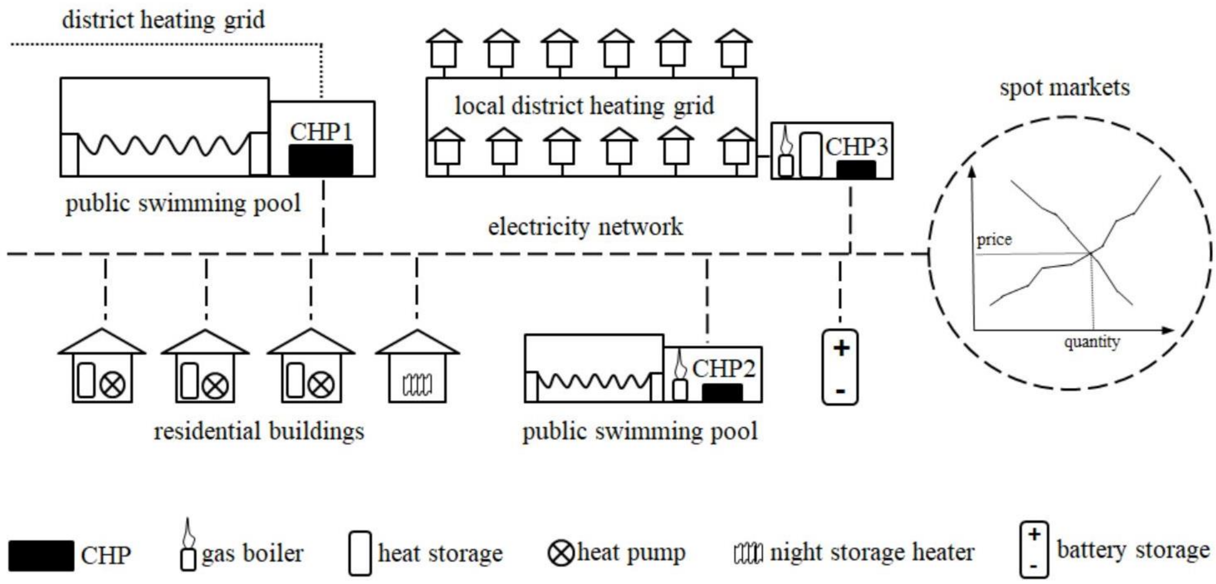

Figure 9: Schematic overview of the case study

4.1 CHP systems based on internal combustion engines

Three CHP systems using gas-fired internal combustion engines are part of the portfolio. System No. 1 provides heat for a large public swimming pool. Here, a district heating grid can deliver alternative heat supply; yet, the CHP system is not designed for feeding back heat to the grid. System No. 2 produces heat for a small public swimming pool and No. 3 is connected to a local heat network with 18 households. For systems No. 2 and 3, gas boilers serve as backup and/or as suppliers for peak heat demands.

Our modelling approach takes into account the basic technical nameplate features such as minimum and maximum generation capacities, fuel efficiencies, and (for CHP units) the CHP coefficients in terms of a constant power-to-heat ratio as well as minimum operation (one hour)

[19] The research project was carried out between 2013 and 2018 in the German state of North-Rhine Westphalia. It was investigated how existing but unused storage facilities in cities can be flexibly operated in practice and which technical and regulatory hurdles have to be overcome to achieve this. During a one year field-test, a portfolio of power producers, consumers and storage devices was controlled according to optimized schedules. Special focus was laid on the development of a modeling, optimization and forecasting environment.

and shut-down times (one quarter hour). Regarding thermal storages, a simplified procedure is used to determine the relevant technical parameters: For the swimming pools, storage capacities are derived from the pool volume, the specific heat capacity of water and from a maximum permitted water temperature variation of ± 0.5 °K. This is assumed to be acceptable from a customers' point of view. Furthermore, it is assumed that thermal storage losses contribute to the overall heating of the swimming pool and therefore, effective storage losses are zero. In contrast, storage losses are factored in for CHP system No. 3 where a 3,000 litres conventional hot water storage unit is connected to a small local heating network. It is noteworthy that network losses are assumed to be constant and therefore are part of the heating demand. Also, for CHP system No. 3, a variation of the heat network´s operating temperature that may provide additional storage capacity is not foreseen within our modelling approach.

Table 3 provides an overview of the key technical parameters of the CHP systems.

Table 3: Key technical parameters of the CHP systems

		CHP system		
Technical parameter		**No. 1**	**No. 2**	**No. 3**
Max. | Min. electrical power CHP unit	[kW_{el}]	420 | 210[20]	50 | 25	19 | 10
Power to heat ratio CHP unit	[-]	0.78	0.63	0.61
Fuel efficiency CHP unit	[%]	86.8	87.8	92.6
Fuel type CHP unit	[-]	natural gas	natural gas	biogas
Max. | Min. thermal power boiler	[kW_{th}]	1,000* | 0	460 | 0	2 x 170 | 0
Fuel efficiency boiler	[%]	100*	90	96
Fuel type boiler	[-]	district heat*	natural gas	natural gas
Thermal storage capacity	[kWh_{th}]	4,946	791	81
Thermal storage efficiency	[%]	100	100	95
Annual heating demand	[MWh]	3,518	1,233	648

* Alternative/peak supply from district heating grid

The relevant economic parameters for the CHP systems are shown in Table 4. Prices for fuel and district heating supply are derived from individual contracts, based on net values and valid

[20] Within our implemented modelling approach, minimum power generation of CHP system No.1 has been set to zero. When submitting linearized bidding curves to the DA and IDO auctions, as described in Section 3.4, it is possible (however not very likely) that an electricity amount is marketed (after consideration of actual price outcomes) that is not physically feasible in the dispatch optimization, i.e. due to minimum production restrictions preventing a convex production range. In a real-world scenario, such a position may still be corrected in the continuous intraday market. As we neglect this market in our approach, we decided to neglect the minimum production capacity of the biggest CHP unit in our portfolio to ensure feasibility.
The optimization triggered penalties associated to slack variables in the electric balance equation (due to over-production) several times during the yearly runs before this relaxation was introduced. However, the effects on total calculated contribution margins by this relaxation should be limited – and affect different scenario cases in a similar manner.

for the modelling period under consideration. Differences in prices also result from different fuel-taxation rules, applied under the German law for the operation of CHP units and gas boilers. Additionally, specific regulatory issues come into effect on the revenue side: Because CHP unit No. 3 is fired with biogas, the Renewable Energy Act (EEG) guarantees a feed-in remuneration as a premium on top of spot electricity prices: The remuneration is calculated retrospectively for each month as the difference between the feed-in tariff guaranteed by law (here: 224.2 €/MWh) and the average of the hourly spot market prices (monthly market value). However, the actual operator revenues result from observed market prices during feed-in with the premium being added on top. Thus, if operation focuses on hours with prices above the average, revenues can exceed the feed-in tariff. Consequently, incentives are given to produce electricity in times of scarcity which is the intention of this remuneration mechanism. Furthermore, CHP units No. 1 and No. 2 receive a compensation from the grid operator for avoided grid cost because it is stipulated in the German regulations that electricity production from distributed CHP units is consumed locally and therefore costs for operating and extending upstream grids are avoided.

Table 4: Key economical parameters of the CHP systems

		CHP system		
Economical parameter (net values)		**No. 1**	**No. 2**	**No. 3**
Fuel price CHP unit	[€/MWh]	31.6	32.9	77.5*
Fuel price boiler	[€/MWh]	37.9**	38.4	38.9
Revenues from avoided grid fees	[€/MWh]	1.6	7.8	-
Revenues from feed-in remuneration	[€/MWh]	-	-	224.2

*Price for gas from biomass
**Price for alternative/peak supply from district heating grid

4.2 Electric heat pumps and storage heaters

Regarding electric heating systems, our study draws on four exemplary households, supplied by heat pumps and conventional solid-state (magnesite) electric storage heaters. The heat pumps are equipped with water-based thermal storages and a supplementary 9 kW heating rod and they also cover demand for domestic hot water. Furthermore, heat pumps No. 1 and No. 3 are brine-water systems, characterised by a higher efficiency (coefficient of performance, COP) compared to heat pump No. 2 which is an air-water system. Even though all mentioned heating systems are subject to minimum power consumption and minimum operation time constraints during their practical application, those restrictions are not taken into consideration here. This is appropriate given the flexibility of heat pumps to turn on and off within one planning interval

- and thus energy consumption within that interval may take arbitrary values. Table 5 provides an overview of the electric heating systems´ key technical parameters.

Table 5: Key technical parameters of the electric heating systems

Technical parameter		**Heat pump 1**	**Heat pump 2**	**Heat pump 3**	**Electric storage heaters**
Max. power consumption	[kW_{el}]	3.0	5.0	4.4	12.0
Coefficient of performance	[-]	5.4	3.0	5.4	1.0
Thermal storage capacity	[kWh_{th}]	29.0	13.0	28.7	36.0
Thermal storage efficiency	[%]	95.0	95.0	95.0	100.0
Annual heating demand	[MWh]	26	43	50	21

The description of the economic parameters for the electric heating systems is straightforward: Operating cost are defined by hourly and/or quarter hourly spot market prices for electricity consumption according to the dispatch schedules. However, it is important to mention that those prices just reflect procurement cost from a supplier´s perspective. Final operating cost that are billed on the customers´ (households) electricity invoice are significantly higher because margins and risk premiums as well as additional regulated price components such as taxes, grid-charges and other levies are included.

4.3 Battery storage system

In addition to the above-mentioned heating systems, a small grid-connected lithium-ion battery storage system is part of the portfolio. It is a stand-alone configuration which means that neither power generation units like PV installations nor consumers are connected directly to the storage system. Consequently, its full capacity can be used to generate profits from exploiting intertemporal spot market price differences. To reduce model complexity and computation time, we ignore calendric as well as cyclic battery ageing and therefore the battery´s usable capacity is assumed to remain constant. Furthermore, self-discharge is neglected and efficiency rates are considered to be independent from the state of charge and from the operating power during charging or discharging. Table 6 summarizes the specific configuration data of the battery storage system:

Table 6: Key technical parameters of the battery storage systems

Technical parameter		**Battery storage system**
Usable storage capacity	[kWh_{el}]	32.5
Max. charging and discharging power	[kW_{el}]	50
Charging and discharging efficiencies	[%]	92.5

5 Results and Discussion

In this section, the results of the backtesting of our sequence of stochastic programs are discussed. These are based on the methodology described in Section 3 and the portfolio described in Section 4. Besides an analysis based on the original real-world portfolio, we carry out sensitivity analyses with two adjusted portfolio setups to identify the influence of available flexibilities if added to or removed from the given setting.

The following parameters are not altered within the following subsections:

In the k-means algorithm used for scenario reduction of the price path simulations (cf. Section 3.3), k=15 and k=60 are chosen to obtain stochastic representations of the optimization problem. The number of scenarios is increased by two extreme price scenarios (one high, one low) each, leading to 17 or 62 scenarios, respectively, entering the optimization problem. The probability of these extreme scenarios is manually set to 0.001 (0.1%) each and all other reduced scenario probabilities are adjusted proportionally, assuring that the sum of probabilities over all scenarios still amounts to 1.

For the parameterization of the bidding curves mentioned in Section 3.4, further assumptions need to be taken. Firstly, -3,000 €/MWh and 3,000 €/MWh are chosen as the left boundary of the lowest interval and the right boundary of the highest interval, respectively, for both DA and ID opening auctions. Secondly, the chosen number of intervals per bidding curve is 12.

It is also noteworthy to mention that results as shown below compare the portfolio´s contribution margins between the different optimization approaches. To determine these margins, revenues from heat production are added to the respective objective values[21]. The revenues are derived from the opportunity costs of alternative supplies for the heat demands. These are calculated based on the prices for supply from the district heating grid (CHP unit No. 1) and from gas boilers (CHP units No. 2 and No. 3).

The optimization model used for solving all optimization problems, and the daily looping structure for the backtesting described in Section 3.5, are implemented in GAMS and solved by use of the Branch-and-Cut algorithm executed with Cplex solver version 12.6[22] in standard settings.

[21] The objective values in all cases are distinctly negative, mainly because said gains from heat production are not considered within the optimization model, but heat demand stills needs to be satisfied. These revenues only depend on heat demands and heat prices; both cannot be influenced by our short-term marketing decisions and are thus not relevant for the optimization.

[22] The backtesting code was executed on a virtual machine with a 2,6 GHz processor and eight cores.

5.1 Results for the real-world portfolio

For our portfolio described in Section 4, the benefits of stochastic optimization are shown along with the implications of imperfect information and limited planning horizons in Figure 10. Applying rolling optimization horizon with perfect information reduces the annual contribution margin by 3,094 € compared to the integral optimization with perfect information. In the examined case, our results indicate that imperfect information induces rather limited additional losses relative to the overall contribution margins, the $EVPI^{RH}$ is 1,746 €. If uncertain prices are modelled stochastically, this foregone value of the expected value problem can be reduced substantially in relative terms. However, the benefits of stochastic optimization are limited in absolute numbers. The BSO_{17}^{RH} for the 17 scenario case amounts to 644 €, or 36.9% of the $EVPI^{RH}$, in the deterministic case. With 62 scenarios, the BSO_{62}^{RH} increases by further 193 € to 837 €, or 47.9% of the $EVPI^{RH}$, in the deterministic case.

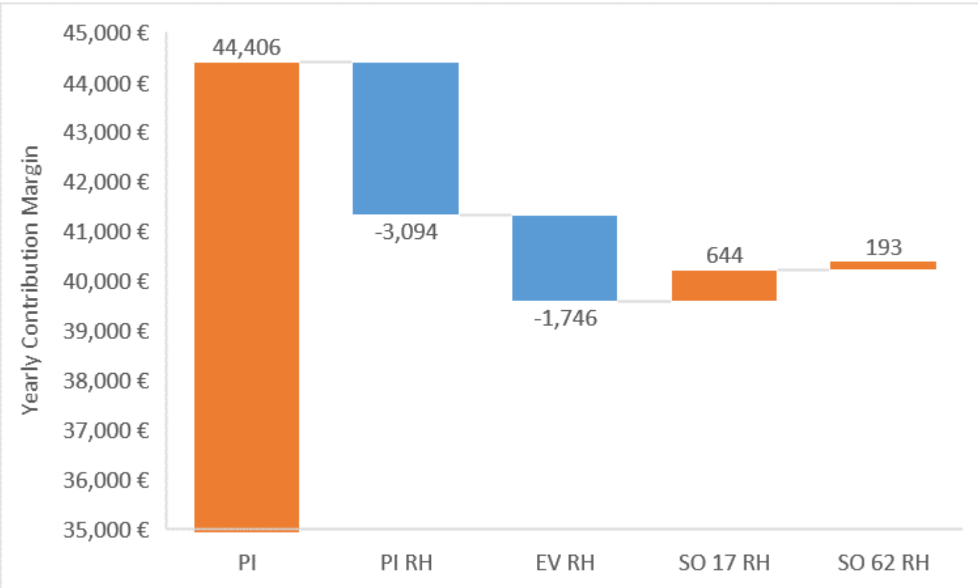

Figure 10: Overview of value added by stochastic optimization

A closer look at the schedules of the electricity producing CHP plants, which mostly drive overall portfolio results, reveals that a problem representation with a higher number of scenarios leads to an adjustment in full-load hours towards the PI^{RH} solution, see Table 7. A higher number of scenarios thus leads to bidding curves coping better with price uncertainty. Notably the cost-optimal heat supply technology for the different heating grids and the corresponding production schedules are identified more accurately. Yet the CHP units under consideration in our case study were designed to achieve high full-load hours, providing heat more or less in base-load operation. Therefore the potentials for an adjustment to variable and uncertain prices remain limited. The CHP unit No. 3, which is remunerated under the renewable energy support scheme, has the highest full-load hours and would operate even more often, if the plant electricity output was not restrained by heat demand. Besides high heat demand, also low fuel costs favour CHP systems compared to pure heating technologies like back-up heat boilers (cf. Table 4) contributing also to this high number of full-load hours. The results for CHP plants No. 2 and 3 show that an improved consideration of price uncertainty leads to a higher share of heat demand covered by CHP, which in turn implies higher full-load hours for CHP No. 2 and No. 3. For CHP No. 1, where the potentials to shift heat production intertemporally are more pronounced due to a larger heat storage, the effect on full-load hours is more ambiguous. But throughout the absolute difference between full-load hours in the SO_n and PI^{RH} cases decreases, the more price uncertainty is captured by an increasing number of scenarios n. Interestingly, the full-load hours are lower in the case with perfect information without a rolling horizon (PI), than with a rolling horizon (PI^{RH}), as storages in the system are now able to shift heat and electricity generation between different days, enabling CHP units to operate in hours with higher electricity prices.

Table 7: Full-load hours of the CHP units in the portfolio with and without perfect information

CHP Unit	PI	PI^{RH}	EV^{RH}	SO_{17}	SO_{62}
No. 1	4,781	5,158	4,815	5,172	5,149
No. 2	7,371	7,691	7,472	7,573	7,585
No. 3	8,250	8,311	8,275	8,277	8,284

Note: the capacity factor of the units may be obtained by dividing the number of full load hours by 8760 h

While this implies a financial improvement through the use of stochastic optimization, the computational cost, however, are increasing disproportionally, as shown in Table 8.[23] On average, 17 scenarios still enable a single solve within minutes, a backtesting procedure for a one year period terminates within a few days. With 62 scenarios, single computation times exceed the relevant up-front optimization times in actual market environments, i.e. an optimization run between DA and IDO auctions will often take longer than the actual bid preparation time (latest announcement time of DA results, 1:50 p.m., closing time of IDO auction 3:00 p.m.). Backtesting times also become poorly manageable, making it hard to assess the quality of the given uncertainty modelling approach within a reasonable time.

[23] The calculations were run on two different (remote) virtual machine servers with the same basic features (2.6 GHz, 8 core processor, 24 GB RAM). The problems were solved in GAMS by CPLEX solver version 12.6.1.0 in default settings (i.e. duality gap =1%). The single perfect information optimization without rolling horizon is not listed in this table. This full-year optimization took 41 minutes and 4 seconds.

Table 8: Runtimes of 1,095 subsequent optimizations with different scenario numbers (times in d:hh:mm:ss)

Number of scenarios n	**Average time per optimization**	**Average time per day (3 optimizations)**	**Total time (1,095 optimizations)**
1 (PI^{RH})	0:00:00:03	0:00:00:08	0:00:49:37
1 (EV^{RH})	0:00:00:03	0:00:00:08	0:00:50:12
17	0:00:02:45	0:00:08:15	2:02:09:00
62	0:01:47:12	0:05:21:36	81:12:25:54

Therefore, the increases in profit and computation time are obviously not well-balanced. The mismatch in our example may be summarized as follows: An improvement in recovered $EVPI^{RH}$ by 11 percentage points (for 62 scenarios, compared to the deterministic case) was paid for with a fortyfold increase of computation time.

5.2 Sensitivity analyses

To further assess the robustness of the obtained results, several assumptions were modified to assess their impact on $EVPI^{RH}$, BSO_n^{RH}, unit operation and on the computational burden.

First, the effect of removing flexibility from the system was assessed. The simplest flexibility option that could be removed without provoking an $EVPI^{RH}$ of close to zero due to binding heating constraints is the battery storage system described in Subsection 4.3. As a result, the contribution margins of the PI^{RH} solution drops by 631 € - this effect is much more pronounced in integral planning (minus 1,500 €). Without this source of flexibility, both the $EVPI^{RH}$ and the BSO_n^{RH} also drop, as can be seen in Figure 11. Expressed in relative numbers, however, the BSO_n^{RH} only decreases slightly, to 35.6% and 43.7%, as opposed to 36.9% and 47.9% in the initial setting.

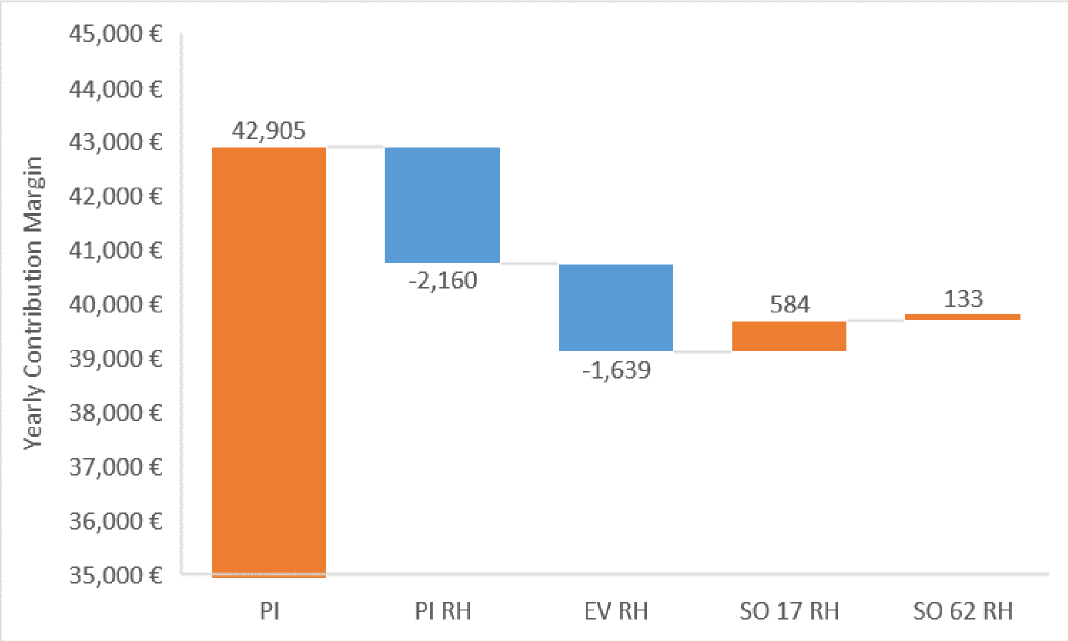

Figure 11: Sensitivity analysis of value added by stochastic optimization without battery storage

As a lack of flexibility from electric storage induces less flexibility in trading decisions and thus a higher importance of individual unit-commitment decisions, computation times for multi-scenario cases go up significantly, as listed in Table 9.

Table 9: Runtimes for the sensitivity analysis without battery storage (times in d:hh:mm:ss)

Number of scenarios n	Average time per optimization	Average time per day (3 optimizations)	Total time (1,095 optimizations)
1 (PI^{RH})	0:00:00:03	0:00:00:08	0:00:47:15
1 (EV^{RH})	0:00:00:02	0:00:00:06	0:00:38:59
17	0:00:03:49	0:00:11:28	2:21:44:12
62	0:03:20:49	0:10:02:27	152:16:55:42

As a second sensitivity, the impact of a higher flexibility potential is investigated. Therefore, the heat demand for the various heating grids is scaled down so that the CHP units would reach 4,380 heat full-load hours in the investigated year, if no alternative heat source was used. This increases the leeway for shifting heat production between hours of the day by means of the installed heat storages. The absolute annual heat demands and their relative changes compared to the original case are displayed in Table 10.

Table 10: Sensitivity analysis with reduced annual heat demands

		CHP No. 1	CHP No. 2	CHP No. 3
Annual Heat Demand	[MWh]	2,394	356	148
Change	[%]	-31.9%	-71.1%	-77.2%

The absolute value of the overall contribution margins decreases significantly given the reduced need for heat generation. In contrast, the gap between the margins with perfect foresight with and without rolling horizon slightly increases, as more heat production can be shifted between subsequent days, if there is no limited optimization time horizon. The results indicate a minor effect on BSO_n^{RH} and $EVPI^{RH}$, respectively. Although the absolute value of the contribution margins has decreased, the effects of perfect information and stochastic modelling of uncertain prices remain on a similar absolute level, as can be seen in Figure 12. While it is arguable whether the $EVPI^{RH}$ and BSO_n^{RH} are thus higher in relative terms regarding the overall portfolio margins, the BSO_n^{RH} relative to the $EVPI^{RH}$ shows a decrease from 36.9% and 47.9% to 28.6% and 32.3%.

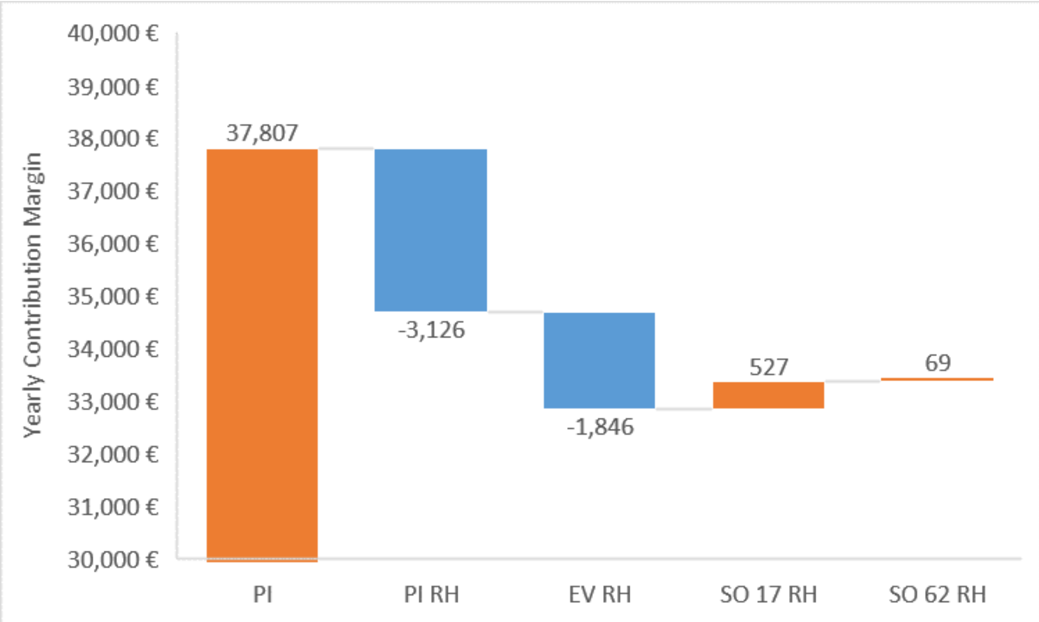

Figure 12: Sensitivity analysis of value added by stochastic optimization with reduced overall heat demand.

While the effects on $EVPI^{RH}$ and BSO_n^{RH} seem ambiguous, the effects on full-load-hours and computation time are evident. Table 11 shows a clear decline in full-load-hours by the reduced heat demand, as intended in this sensitivity analysis. As before, a higher number of scenarios induces a converging behaviour of full-load hours towards the PI^{RH} solution. As before, the full-load-hours slightly decrease, when there is no limited optimization time-horizon, indicating more efficient heat storage usage between subsequent days.

Table 11: Full-load hours of CHP units for sensitivity analysis with reduced annual heat demands

CHP Unit	PI	PI^{RH}	EV^{RH}	SO_{17}	SO_{62}
No.1	4,042	4,131	3,901	4,084	4,078
No. 2	4,338	4,344	4,328	4,329	4,322
No. 3	6,400	6,613	6,520	6,596	6,550

Computation times, however, rise significantly by this addition of leeway, as can be seen in Table 12. For 17 scenarios, calculation times on average quadruple, as less dispatch decisions are determined by high heat demand periods. Therefore, the optimization model needs more time to decide on the binary variable values determining the operation state of the CHP units.[24]

Table 12: Runtimes for the sensitivity analysis with reduced annual heat demands (times in d:hh:mm:ss)

Number of scenarios n	Average time per optimization	Average time per day (3 optimizations)	Total time (1,095 optimizations)
1 (PI^{RH})	0:00:00:03	0:00:00:08	0:00:50:48
1 (EV^{RH})	0:00:00:03	0:00:00:08	0:00:50:48
17	0:00:12:44	0:00:38:11	9:16:14:50
62	0:04:49:51	0:14:29:34	220:09:53:25

6 Conclusion

In this paper, we investigate the benefits of stochastic programming in the context of the operation of heat supply technologies in small-scale district heating grids with the example of a real-world portfolio. We model the uncertainty of quarter-hourly electricity prices based on an approach by Pape, et al. (2017), originally designed for hourly products. Uncertainty in heat demand is not considered. Price scenario paths are created using Monte Carlo Simulation. These paths are subsequently reduced by applying a k-means algorithm. Furthermore, we consider bidding curve characteristics as specified notably by the European EPEXSPOT market, directly in our multistage portfolio optimization model, making use of a novel approach to choose the bidding curve's supporting points. The model formulation furthermore implements different cogeneration, heat conversion, and storage technologies.

We furthermore suggest a concept for the evaluation of benefits from stochastic optimization in rolling-horizon applications, i.e. the expected value of perfect information with a rolling horizon ($EVPI^{RH}$) and the benefit of stochastic solution with a rolling horizon and n scenarios (BSO_n^{RH}).

[24] This increasing computational burden is significantly less pronounced in the full-year optimization with perfect foresight, as the duration increases to 40 minutes and 23 seconds, thus by only 19 seconds compared to our initial setting.

We find that under the given assumptions and modelling choices, stochastic optimization improves the contribution margins of the modelled portfolio. It is possible to recover a significant share (>35%) of the $EVPI^{RH}$ that would be lost in a deterministic recourse representation of the problem. Computation time, however, is rising disproportionally, negatively impacting the attractiveness of implementation of high scenario representations. In the given setting, the overall absolute BSO_n^{RH} is limited compared to the overall contribution margins. By removing or adding inter-temporal flexibilities through storages on both the electric and the heat side of the portfolio, these results do not change significantly.

The research conducted in this paper confirms the repeatedly described benefits of stochastic modelling of uncertainty. However, we show that the performance of stochastic optimization approaches depends highly on technical characteristics of the units involved and, even more, on the rigidity induced by equality constraints such as heat demand constraints. As the CHP units under study have a fixed power-to-heat ratio and short-term optimal dispatch of storage systems is not necessarily influenced by more accurate price predictions, stochastic optimization faces limitations.

Finally, we find that stochastic optimization in this setting suffers heavily from the curse of dimensionality, if scenario numbers are increased. A high number of equality restrictions and binary unit-commitment variables strongly increase the computational burden, limiting the applicability in real-world contexts with high scenario numbers. However, also smaller numbers of scenarios prove to provide useful decision support for real world applications.

Acknowledgements

This work partly was supported by the German Federal Ministry of Economic Affairs and Energy on the basis of a decision by the German Bundestag [grant number 0325527].

Supported by:

Federal Ministry
for Economic Affairs
and Energy

on the basis of a decision
by the German Bundestag

References

Ackermann, S., Paulus, S., Steinke, F. & Szabo, A., 2019. Comparison of Two Day-Ahead Offering Strategies for a Flexible CHP Plant in Germany. *IEEE PES Innovative Smart Grid Technologies Europe*.

Birge, J. & Louveaux, F., 2011. The value of Information and the Stochastic Solution. In: T. Mikorsch, S. Resnick & S. Robinson, Hrsg. *Introduction to Stochastic Programming - Second Edition*. s.l.:Springer, pp. 163-177.

Böhringer, M. et al., 2019. Trading Strategy for a Flexible Factory Participating in the German Balancing and Day-Ahead Market. *54th International Universities Power Engineering Conference*.

Boomsma, T. K., Juul, N. & Fleten, S.-E., 2014. Bidding in sequential electricity markets: The Nordic case. *European Journal of Operational Research*, pp. 797-809.

Brand, H. & Weber, C., 2005. *Deriving optimal bid functions taking into account inter-temporal restrictions.* Liège, PSCC.

Bundesnetzagentur, 2011. *Beschluss Az: BK6-10-097 Festlegungsverfahren zu den Ausschreibungsbedingungen und Veröffentlichungspflichten für Primärregelleistung.* [Online] Available at: https://www.bundesnetzagentur.de/DE/Service-Funktionen/Beschlusskammern/1_GZ/BK6-GZ/2010/2010_0001bis0999/2010_001bis099/BK6-10-097bis-099/BK6-10-097_Beschluss_2011_04_12.pdf;jsessionid=311973B5761BF1AEDFBDF545EB79C553?__blob=publicationFile&v=3
[Zugriff am 12 September 2019].

Bundesnetzagentur, 2011. *Beschluss Az: BK6-10-098 Festlegungsverfahren zu den Ausschreibungsbedingungen und Veröffentlichungspflichten für Sekundärregelleistung.* [Online] Available at: https://www.bundesnetzagentur.de/DE/Service-Funktionen/Beschlusskammern/1_GZ/BK6-GZ/2010/2010_0001bis0999/2010_001bis099/BK6-10-097bis-099/BK6-10-098_Beschluss_2011_04_12.pdf?__blob=publicationFile&v=2
[Zugriff am 12 September 2019].

Bundesnetzagentur, 2011. *Beschluss Az: BK6-10-099.* [Online] Available at: https://www.bundesnetzagentur.de/DE/Service-Funktionen/Beschlusskammern/1_GZ/BK6-GZ/2010/2010_0001bis0999/2010_001bis099/BK6-

10-097bis-099/BK6-10-099_Beschluss_2011_10_18.pdf?__blob=publicationFile&v=2
[Zugriff am 12 September 2019].

De Ridder, F. & Claessens, B., 2014. A trading strategy for industrial CHPs on multiple power markets. *International Transactions on Electrical Energy Systems*, pp. 677-697.

Dietrich, A., 2020. Incentives for flexible consumption and production of electricity on end-user level - Evidence from a German case study and outlook for 2030. *Unpublished Working Paper*.

Dietrich, A. et al., 2018. *Gemeinsamer Abschlussbericht des Forschungsvorhabens "Die Stadt als Speicher" - Energietechnische u. -wirtschaftl. Bündelung vielfältiger lokaler Speicherkapazitäten innerhalb städtischer Lastzentren zum Ausgleich der Fluktuation erneuerbarer Einspeiser,* s.l.: s.n.

EPEXSPOT, 2019. *EPEX Spot Operational Rules.* [Online] Available at: http://www.epexspot.com/en/extras/download-center/documentation
[Zugriff am 2 July 2019].

EPEXSPOT, 2019. *ETS Hour bidding form - DE/LU.* [Online] Available at: http://www.epexspot.com/en/extras/download-center/trading_documents
[Zugriff am 1 July 2019].

EPEXSPOT, 2020. *EPEX SPOT and ECC successfully launch Intraday auctions in Austria, Belgium, France and the Netherlands.* [Online] Available at: https://www.epexspot.com/en/news/epex-spot-and-ecc-successfully-launch-intraday-auctions-austria-belgium-france-and-netherlands
[Zugriff am 3 December 2020].

Fleten, S.-E. & Kristoffersen, 2007. Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer. *European Journal of Operational Research*, pp. 916-928.

Frances, D. & Kwon, R. H., 2012. Optimization-Based Bidding in Day-Ahead Electricity Auction Markets: A Review of Models for Power Producers. In: S. e. al., Hrsg. *Handbook of Networks in Power Systems I, Energy Systems.* s.l.:Springer-Verlag Berlin Heidelberg.

Furtwängler, C. & Weber, C., 2019. Spot and reserve market equilibria and the influence of new reserve market participants. *Energy Economics*, pp. 408 - 421.

Han, X., Kardakos, E. G. & Hug, G., 2017. Trading strategy for decentralized energy resources in sequential electricity markets: A Swiss case study. *2017 IEEE Innovative Smart Grid Technologies - Asia* .

Havel, P. & Simovic, T., 2013. Optimal planning of cogeneration production with provision of ancillary services. *Electric Power System Research*, Issue 95, pp. 47-55.

Kath, C. & Ziel, F., 2018. The value of forecasts: Quantifying the economic gains of accurate quarter-hourly price forecasts. *Energy Economics*, 17 Oktober, Issue 76, pp. 411-423.

Kempgens, M., 2018. *Strategien für die Elektromobilität bei erweiterter energiewirtschaftlicher Nutzung - Betrachtung steuernder Ladestrategien für Elektrofahrzeuge sowie Minutenreservevermarktung von Elektrofahrzeugen.* Essen: s.n.

Kumbartzky, N., Schacht, M., Schulz, K. & Werners, B., 2017. Optimal operation of a CHP plant participating in the German electricity balancing and day-ahead spot market. *European Journal of Operational Research*, pp. 390-404.

Lloyd, S., 1982. Least Squares Quantization in PCM. *IEEE Transactions on Information Theory*, March, pp. 129-137.

MacQueen, J., 1967. Some methods for classification and analysis of multivariate observations. *Proceedings of the Berkeley Symposium on Mathematical Statistics and Probability*, pp. 281-297.

Pape, C., Biegler-König, R. & Weber, C., 2016. Erstellung einer viertelstündlichen Price Forward Curve für elektrische Energie. *Energiewirtschaftliche Tagesfragen (1/2)*, pp. 50-53.

Pape, C., Vogler, A., Woll, O. & Weber, C., 2017. Forecasting the distributions of hourly electricity spot prices – accounting for serial correlation patterns and non-normality of price distributions. *HEMF Working Paper*, Issue 05/2017.

Schulz, K. & Werners, B., 2015. Capacity determination of ultra-long flexibility investments for district heating systems. *Journal of Business Economics*, pp. 663-692.

Sen, S., Yu, L. & Genc, T., 2006. A Stochastic Programming Approach to Power Portfolio Optimization. *Operations Research*, January-February, 54(1), pp. 55-72.

Thorin, E., Brand, H. & Weber, C., 2005. Long-term optimization of cogeneration systems in a competitive market environment. *Applied Energy*, 81(2), pp. 152-169.

Woll, O. & Weber, C., 2006. *Valuation of CHP Power Plant Portfolios Using Recursive Stochstic Optimization.* Stockholm, 9th International Conference on Probabilistic Methods Applied to Power Systems.

Zapata Riveros, J., Bruninx, K., Poncelet, K. & D'haeseleer, W., 2015. Bidding strategies for virtual power plants considering CHPs and intermittent renewables. *Energy Conversion and Management*, pp. 408-418.

Appendix 1: Portfolio Optimization Model

This section contains the full description of the portfolio optimization model introduced in Section 3.4. The implementation of this MILP model in GAMS consists of an objective function and about 50 further types of equality and inequality constraints. For the sake of simplicity, some constraints are hereby displayed jointly that are represented by more than one constraint in the actually implemented model.

Appendix 1.1: Nomenclature

The tables below list all relevant indices, sets, variables and parameters as used in the model description.

Table 13: Nomenclature for indices

Indices	**Description**
n	scenario node
h	hour
t	quarter hour
u	power/heat unit
hs	heating system
i	intervals for quarter-hourly bidding curve
ih	intervals for hourly bidding curve
det	deterministic

Table 14: Nomenclature for sets

Index sets	**Description**
N	Set of all scenarios n
U	Set of all power or heat producing units u
$U^{boiler} \in U$	Set of all heat boilers, subset of U
$U^{heatstorage} \in U$	Set of all heat storages, subset of U
$U^{batterystorage} \in U$	Set of all battery storages, subset of U
$U^{chp} \in U$	Set of all combined heat and power units, subset of U
$U^{heatpump} \in U$	Set of all heat pump units, subset of U
NH	Set of consistent hour/scenario combinations (n, h)
NT	Set of consistent quarter-hour/scenario combinations (n, t)
HT	Set of consistent quarter-hour/hour mappings (h, t)
HS	Set of all heating systems hs

TI	Mapping of quarter-hourly price simulations $P_{n,t}^{ID}$ to intervals for quarter-hourly bidding curve i for each scenario n
HIH	Mapping of hourly price simulations $P_{n,h}^{DA}$ to intervals for hourly bidding curve ih for each scenario n
T_{det}	Quarter-hourly time-steps t belonging to the deterministic time-horizon of the stochastic optimization
H_{det}	Hourly time-steps h belonging to the deterministic time-horizon of the stochastic optimization

Table 15: Nomenclature for variables

Variables	Unit	Range	Description
$p_{n,h}^{DA}$	kW_{el}	$\mathbb{R}$	Net marketed power in DA Auction per hour h in scenario n
$p_{n,t}^{ID}$	kW_{el}	$\mathbb{R}$	Net marketed power in ID Auction per quarter hour t in scenario n
$p_{n,u,t}^{Power}$	kW_{el}	$\mathbb{R}_0^+$	Produced power by unit u in quarter hour t in scenario n
$q_{n,u,t}^{Fuel}$	kW_{th}	$\mathbb{R}_0^+$	Used fuel by unit u in quarter hour t in scenario n
$q_{n,t}^{DHeat}$	kW_{th}	$\mathbb{R}_0^+$	District heat obtained from the district heating grid in quarter hour t in scenario n
$q_{n,u,t}^{Heat}$	kW_{th}	$\mathbb{R}$	Produced heat by unit u in quarter hour t in scenario n (may be negative for charging heat storages)
$vol_{n,u,t}^{Heat}$	kWh_{th}	$\mathbb{R}_0^+$	Heat storage filling level by unit u in quarter hour t in scenario n
$p_{n,u,t}^{Charge}$	kW_{el}	$\mathbb{R}_0^+$	Power charged to battery storage u in quarter hour t in scenario n
$p_{n,u,t}^{Discharge}$	kW_{el}	$\mathbb{R}_0^+$	Power discharged from battery storage u in quarter hour t in scenario n
$vol_{n,u,t}^{Power}$	kWh_{el}	$\mathbb{R}_0^+$	Battery storage filling level by unit u in quarter hour t in scenario n
$p_{n,u,t}^{Cons}$	kW_{el}	$\mathbb{R}_0^+$	Electricity consumption by heat pump u in quarter hour t in scenario n
$o_{n,u,t}$	-	$\{0;1\}$	Binary power plant operation variable (1: on, 0: off) for unit u in quarter hour t in scenario n
$up_{n,u,t}$	-	$\{0;1\}$	Binary power plant operation variable (1: plant starting, 0: plant not starting) for unit u in quarter hour t in scenario n
$p_{t,i}^{ID,lb}$	kW_{el}	$\mathbb{R}$	Power amount bid at the left boundary of the quarter hourly price interval i for quarter hour t
$p_{t,i}^{ID,rb}$	kW_{el}	$\mathbb{R}$	Power amount bid at the right boundary of the quarter hourly price interval i for quarter hour t
$p_{h,ih}^{DA,lbh}$	kW_{el}	$\mathbb{R}$	Power amount bid at the left boundary of the hourly price interval ih for hour h

$p_{h,ih}^{DA,rbh}$	kW_{el}	$\mathbb{R}$	Power amount bid at the right boundary of the hourly price interval ih for hour h

Table 16: Nomenclature for parameters

Parameters	Unit	Range	Description
φ_n	-	[0,1]	Scenario probability of scenario n
$P_{n,h}^{DA}$	€/kW_{el}	$\mathbb{R}$	DA Auction Price of hour h in scenario n
$P_{n,t}^{ID}$	€/kW_{el}	$\mathbb{R}$	ID Auction Price of quarter-hour t in scenario n
T_u^{Comp}	€/kW_{el}	$\mathbb{R}_0^+$	EEG compensation payment for produced power of unit u
$C_{n,u,t}^{Fuel}$	€/kW_{th}	$\mathbb{R}_0^+$	Fuel costs (incl. CO_2) for unit u in quarter-hour t in scenario n
$C_{n,t}^{DHeat}$	€/kW_{th}	$\mathbb{R}_0^+$	District heat price in quarter-hour t in scenario n
Δh	h		Duration of one hour (1h)
Δt	h		Duration of one quarter-hour (1/4 h)
D_t^{Power}	kW_{el}	$\mathbb{R}_0^+$	Power demand position in quarter -hour t (from previous marketing in hourly and quarter hourly markets)
D_t^{Heat}	kW_{th}	$\mathbb{R}_0^+$	Local heat demand in quarter hour t
$Q_u^{Heat,max}$	kW_{th}	$\mathbb{R}_0^+$	Maximum heat production capacity of unit u
η_u	-	[0,1]	Boiler efficiency of unit u
$Q_u^{Heat,dischargemax}$	kW_{th}	$\mathbb{R}_0^+$	Maximum heat storage discharge capacity of unit u
$Q_u^{Heat,chargemax}$	kW_{th}	$\mathbb{R}_0^+$	Maximum heat storage charge capacity of unit u
$V_u^{Heat,max}$	kWh_{th}	$\mathbb{R}_0^+$	Maximum heat storage filling level of unit u
$V_u^{Heat,Start}$	kWh_{th}	$\mathbb{R}_0^+$	Heat storage filling start level of unit u
$V_u^{Heat,End}$	kWh_{th}	$\mathbb{R}_0^+$	Heat storage filling end level of unit u
η_u^{Losses}	-	[0,1]	Heat storage efficiency of unit u
$P_u^{Power,dischargemax}$	kW_{el}	$\mathbb{R}_0^+$	Maximum battery storage discharge capacity of unit u
$P_u^{Power,chargemax}$	kW_{el}	$\mathbb{R}_0^+$	Maximum battery storage charge capacity of unit u
$V_u^{Power,max}$	kWh_{el}	$\mathbb{R}_0^+$	Maximum battery storage filling level of unit u
$V_u^{Power,Start}$	kWh_{el}	$\mathbb{R}_0^+$	Battery storage filling start level of unit u
$V_u^{Power,End}$	kWh_{el}	$\mathbb{R}_0^+$	Battery storage filling end level of unit u
η_u^{Charge}	-	[0,1]	Battery storage charging efficiency of unit u
$\eta_u^{Discharge}$	-	[0,1]	Battery storage discharging efficiency of unit u
P_u^{Max}	kW_{el}	$\mathbb{R}_0^+$	Maximum power plant production limit of unit u
P_u^{Min}	kW_{el}	$\mathbb{R}_0^+$	Minimum stable power plant production limit (if power plant is running) of unit u

$P_u^{Cons,Min}$	kW$_{el}$	$\mathbb{R}_0^+$	Minimum heat pump electricity consumption of unit u
$P_u^{Cons,Max}$	kW$_{el}$	$\mathbb{R}_0^+$	Maximum heat pump electricity consumption of unit u
COP_u	kW$_{th}$/kW$_{el}$	[0,1]	Heat pump Coefficient of Performance of unit u
b_u^{bp}	kW$_{th}$/kW$_{el}$	$\mathbb{R}_0^+$	Slope of the backpressure curve (pq-diagram) of unit u
a_u^{bp}	kW$_{th}$	$\mathbb{R}$	Section of the backpressure curve (pq-diagram) of unit u
b_u^{Fuel}	kW$_{th}$/kW$_{el}$	$\mathbb{R}_0^+$	Slope of the fuel consumption curve of unit u
a_u^{Fuel}	kW$_{th}$	$\mathbb{R}_0^+$	Minimal fuel consumption, section of the fuel consumption curve of unit u
OP_u	-	$\mathbb{R}_0^+$	Minimum operation period number of plant u
SD_u	-	$\mathbb{R}_0^+$	Minimum shut-down period number of plant u
$\lambda_{n,t}$	-	[0,1]	Linearization parameter for the position of quarter-hourly spot prices within the quarter-hourly bidding curve in quarter-hour t in scenario n
$\lambda_{n,h}$	-	[0,1]	Linearization parameter for the position of hourly spot prices within the hourly bidding curve in hour h in scenario n

Appendix 1.2: Model formulation

The objective function includes the revenues obtained in the spot markets, further revenue streams through Renewable infeed premia and the costs associated with power and heat generation:

$$\max_{p_{n,h}^{DA},p_{n,t}^{ID},q_{n,u,t}^{Power},q_{n,u,t}^{Fuel},q_{n,t}^{Heat}} \sum_{n\in N} \varphi_n \cdot \Bigg(\sum_{h|(n,h)\in NH} p_{n,h}^{DA} \cdot P_{n,h}^{DA} \cdot \Delta h + \Bigg(\sum_{t|(n,t)\in NT} \Big(p_{n,t}^{ID} \cdot P_{n,t}^{ID} - q_{n,t}^{DHeat} \cdot C_{n,t}^{DHeat} + \sum_{u\in U} p_{n,u,t}^{Power} \cdot T_u^{Comp} - q_{n,u,t}^{Fuel} \cdot C_{n,u,t}^{Fuel} \Big) \Bigg) \cdot \Delta t \Bigg) \tag{3}$$

There are both electric and heat balance equations implemented in the model. The electric balance equation balances the trading position and the physical fulfilment of said position:

$$\forall (n,h) \in NH \wedge \forall (n,t) \in NT \wedge \forall (h,t) \in HT:$$

$$\sum_{u \in U} q_{n,u,t}^{Power} = D_t^{Power} + D_h^{Power} + p_{n,t}^{ID} + p_{n,h}^{DA} \tag{4}$$

For six out of seven local heating systems, the following balance equation holds:

$$\forall (n,t) \in NT \wedge \forall hs \in HS: \sum_{(u)|(u,hs)} q_{n,u,t}^{Heat} + q_{n,t}^{DHeat} = D_t^{Heat} \tag{5}$$

Thereby the option to obtain heat $q_{n,t}^{DHeat}$ from a district heating grid is only relevant for one considered heating system (system with CHP No. 1). For all other systems $q_{n,t}^{DHeat} = 0$.

For heating boilers, the following constraints apply, describing the maximum production capacity (6) and the boiler efficiency (7):

$$\forall (n,t) \in NT, u \in U^{boiler}: q_{n,u,t}^{Heat} \leq Q_u^{Heat,max} \tag{6}$$

$$\forall (n,t) \in NT, u \in U^{boiler}: q_{n,u,t}^{Fuel} = \frac{q_{n,u,t}^{Heat}}{\eta_u} \tag{7}$$

The heat storages are subject to the following constraints, describing maximum storage level changes ((8), (9)), maximum storage level (10), and storage level changes between time-steps (11), as well as start (12) and end storage levels (13):

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: q_{n,u,t}^{Heat} \leq Q_u^{Heat,dischargemax} \tag{8}$$

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: q_{n,u,t}^{Heat} \geq Q_u^{Heat,chargemax} \tag{9}$$

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} \leq V_u^{Heat,max} \tag{10}$$

$$\forall (n,t) \in NT, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} = v_{n,u,t-1}^{Heat} \cdot \eta_u^{Losses} - q_{n,u,t}^{Heat} \cdot \Delta t \tag{11}$$

$$\forall t \in \{1\}, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} = V_u^{Heat,Start} \cdot \eta_u^{Losses} - q_{n,u,t}^{Heat} \cdot \Delta t \tag{12}$$

$$\forall t \in \{240\}, (n,t) \in NT, \forall u \in U^{heatstorage}: v_{n,u,t}^{Heat} = V_u^{Heat,End} \tag{13}$$

Similarly, the battery storage is subject to the following constraints describing maximum storage level changes ((14), (15)), maximum storage level (16), and storage level changes between time-steps (17), as well as start (18) and end storage levels (19):

$$\forall\,(n,t)\;\in\;NT, \forall\,u\;\in U^{batterystorage}\colon p_{n,u,t}^{Power}\;\le\;P_{u}^{Power,dischargemax} \tag{14}$$

$$\forall\,(n,t)\;\in\;NT, \forall\,u\;\in U^{batterystorage}\colon p_{n,u,t}^{Power}\;\ge -\,P_{u}^{Power,chargemax} \tag{15}$$

$$\forall\,(n,t)\;\in\;NT, \forall\,u\;\in U^{batterystorage}\colon v_{n,u,t}^{Power}\;\le\;V_{u}^{Power,max} \tag{16}$$

$$\forall\,(n,t)\;\in\;NT, \forall\,u\;\in U^{batterystorage}\colon v_{n,u,t}^{Power} = v_{n,u,t-1}^{Power} + \left(p_{n,u,t}^{Charge}\cdot\eta_{u}^{Charge} - p_{n,u,t}^{Discharge}\cdot\eta_{u}^{Discharge}\right)\cdot\Delta t \tag{17}$$

$$\forall\,t\in\{1\}, \forall\,u\;\in U^{batterystorage}\colon v_{n,u,t}^{Power} = V_{u}^{Power,Start} + \left(p_{n,u,t}^{Charge}\cdot\eta_{u}^{Charge} - p_{n,u,t}^{Discharge}\cdot\eta_{u}^{Discharge}\right)\cdot\Delta t \tag{18}$$

$$\forall\,t\in\{240\}, (n,t)\;\in\;NT, \forall\,u\;\in U^{batterystorage}\colon v_{n,u,t}^{Power} = V_{u}^{Power,End} \tag{19}$$

For all electric units also delivering heat $U^{chp}\in U$, the maximum heat constraint is given by:

$$\forall\,u\;\in U^{chp}\colon\; q_{n,u,t}^{Heat}\le Q_{u}^{Heat,max} \tag{20}$$

The limits for electricity production are given by:

$$\forall\,u\;\in U^{chp}\colon\; p_{n,u,t}^{Power}\le o_{n,u,t}\cdot P_{u}^{Max} \tag{21}$$

$$\forall\,u\;\in U^{chp}\colon\; p_{n,u,t}^{Power}\ge o_{n,u,t}\cdot P_{u}^{Min} \tag{22}$$

For all combined heat and power producing motor units $u \in U^{chp}$ in the system, the following equations hold to describe the relationship of produced electricity and produced heat (also often denoted as pq-diagram, (23)), as well as the CHP fuel consumption depending on electricity production (24):

$$\forall\, u \in U^{chp}:\ q_{n,chp,t}^{heat} = a_{chp}^{bp} \cdot o_{n,chp,t} + b_{u}^{bp} \cdot p_{n,chp,t}^{elec} \tag{23}$$

$$\forall\, u \in U^{chp}:\ q_{n,chp,t}^{Fuel} = a_{chp}^{Fuel} \cdot o_{n,chp,t} + b_{chp}^{Fuel} \cdot p_{n,chp,t}^{elec} \tag{24}$$

For the heat pump units $u \in U^{heatpump}$, the following restrictions hold to describe minimum (25) and maximum heat pump electricity consumption (26), as well as the electricity/heat conversion efficiency (27) and the consideration of electricity consumed in the overall electricity balance (28):

$$\forall\, u \in U^{heatpump}:\ p_{n,u,t}^{Cons} \geq o_{n,u,t} \cdot P_{u}^{Cons,Min} \tag{25}$$

$$\forall\, u \in U^{heatpump}: p_{n,u,t}^{Cons} \leq o_{n,u,t} \cdot P_{u}^{Cons,Max} \tag{26}$$

$$\forall\, u \in U^{heatpump}: q_{n,u,t}^{Heat} = p_{n,u,t}^{Cons} \cdot COP_{u} \tag{27}$$

$$\forall\, u \in U^{heatpump}: p_{n,u,t}^{Cons} = -p_{n,u,t}^{Power} \tag{28}$$

The minimum operation time and minimum shut down times of the CHP motors are modelled by use of further binary variables and by introducing further time-coupling constraints, which describe minimum operation time (29), minimum shut-down time (30) and the definition of a power plant start (31):

$$\forall\, (n,t) \in NT, \quad (n,t-1) \in NT, u \in U^{chp}: \sum_{t'=t-OP_u+1}^{t} up_{n,u,t'} \leq o_{n,u,t} \tag{29}$$

$$\forall\, (n,t) \in NT, (n,t-1) \in NT, u \in U^{chp}: \sum_{t'=t-SD_u+1}^{t} up_{n,u,t'} \leq 1 - o_{n,u,t-SD_u} \tag{30}$$

$$\forall\, (n,t) \in NT, (n,t-1) \in NT, u \in U^{chp}: up_{n,u,t} \geq o_{n,u,t} - o_{n,u,t-1} \tag{31}$$

Spot trading according to EPEXSPOT market rules is realized by the following constraints:

Forbidden marketing in the deterministic part of the optimization ((34), (37)), as well as minimum ((32), (35), (38)) and maximum ((33), (36), (39)) marketing amounts for each interval of the bidding curve for Day Ahead Auction ((32), (33), (34)), Intraday Opening Auction ((35), (36), (38)) and Combined marketing to both auctions ((38), (39)) are described by the following restrictions:

$$\forall\, (t,i) \in TI:\; p_{t,i}^{ID,lb} \geq -\sum_{u\, \in U^{batterystorage}} P_u^{Power,chargemax} - \sum_{u\, \in U^{heatpump}} P_u^{Cons,Max} \tag{32}$$

$$\forall\, (t,i) \in TI:\; p_{t,i}^{ID,rb} \leq -\sum_{u\, \in U^{batterystorage}} P_u^{Power,dischargemax} + \sum_{u} P_u^{Max} \tag{33}$$

$$\forall\, t \in T_{det}:\; p_{n,t}^{ID} = 0 \tag{34}$$

$$\forall\, (h,ih) \in HIH:\; p_{h,ih}^{DA,lbh} \geq -\sum_{u\, \in U^{batterystorage}} P_u^{Power,chargemax} - \sum_{u\, \in U^{heatpump}} P_u^{Cons,Max} \tag{35}$$

$$\forall\, (H,ih) \in HIH:\; p_{h,ih}^{DA,rbh} \leq -\sum_{u\, \in U^{batterystorage}} P_u^{Power,dischargemax} + \sum_{u} P_u^{Max} \tag{36}$$

$$\forall\, h \in H_{det}:\; p_{n,h}^{DA} = 0 \tag{37}$$

$$\forall\, (h,ih) \in HIH \wedge (h,t) \in HT:\; p_{h,ih}^{DA,lbh} + p_{t,i}^{ID,lb} \geq -\sum_{u\, \in U^{batterystorage}} P_u^{Power,chargemax} - \sum_{u\, \in U^{heatpump}} P_u^{Cons,Max} \tag{38}$$

$$\forall\, (h,ih) \in HIH \wedge (h,t) \in HT:\; p_{h,ih}^{DA,rbh} + p_{t,i}^{ID,rb} \leq -\sum_{u\, \in U^{batterystorage}} P_u^{Power,dischargemax} + \sum_{u} P_u^{Max} \tag{39}$$

The following restrictions model the linearization of the optimized bidding curves within the bidding curve intervals and are defined for both the Day Ahead ((43),(44),(45)) and Intraday Opening Auction ((40),(41),(42)) bidding curves:

$$\forall\,(t,i) \in TI \;\wedge\; (n,t) \in NT:\; p_{n,t}^{ID} = \left(1-\lambda_{n,t}\right) \cdot p_{t,i}^{ID,lb} + \lambda_{n,t} \cdot p_{t,i}^{ID,rb} \tag{40}$$

$$\forall\,(n,t) \in NT:\; p_{t,i}^{ID,lb} \leq p_{t,i}^{ID,rb} \tag{41}$$

$$\forall\,(t,i) \in TI, i < I:\; p_{t,i}^{ID,rb} = p_{t,i+1}^{ID,lb} \tag{42}$$

$$\forall\,(h,ih) \in HIH \;\wedge\; (n,h) \in NH:\; p_{n,h}^{DA} = \left(1-\lambda_{n,h}\right) \cdot p_{h,ih}^{DA,lbh} + \lambda_{n,h} \cdot p_{h,ih}^{DA,rbh} \tag{43}$$

$$\forall\,(n,h) \in NH:\; p_{h,ih}^{DA,lbh} \leq p_{h,ih}^{DA,rbh} \tag{44}$$

$$\forall\,(h,ih) \in HIH, ih < IH:\; p_{h,ih}^{DA,rbh} = p_{h,ih+1}^{DA,lbh} \tag{45}$$

Finally, the condition avoiding arbitrage trades between hours and quarter-hours in the 1st Optimization is given by:

$$\forall\,(n,t) \in NT:\; \sum_{(h,t)\in HT} p_{n,t}^{ID} = 0 \tag{46}$$

Correspondence

Dipl.-Kfm. Andreas Dietrich

PhD candidate

House of Energy Markets and Finance

University of Duisburg-Essen, Germany

Berliner Platz 6-8, 45127 Essen

E-Mail andreas.dietrich@uni-due.de

Christian Furtwängler, M.Sc.

PhD candidate/Academic Staff

House of Energy Markets and Finance

University of Duisburg-Essen, Germany

Berliner Platz 6-8, 45127 Essen

Tel. +49 201 183-6458

Fax +49 201 183-2703

E-Mail christian.furtwaengler@uni-due.de

Prof. Dr. Christoph Weber

Chairholder for Management Science and Energy Economics

House of Energy Markets and Finance

University of Duisburg-Essen, Germany

Berliner Platz 6-8, 45127 Essen

Tel. +49 201 183-2966

Fax +49 201 183-2703

E-Mail christoph.weber@uni-due.de